

Round I: Purely Objective, 60 Marks, 2 Hours

Section I - True or False - 10 Questions, 10 Marks (1 Mark Each)

Section II - MCQs - 10 Questions, 20 Marks (2 Marks Each)

Section III - Two-Digit Answers Only - 10 Questions, 30 Marks (3 Marks Each)

Section I : Each question carries 1 mark

विभाग I : प्रत्येक प्रश्नाला 1 गुण आहे

For Q.1 to 10: State whether TRUE or FALSE.

Select the box labelled T if the statement is TRUE.

Select the box labelled F if the statement is FALSE.

प्रश्न क्रमांक 1 ते 10: चूक की बरोबर ते ओळखा.
दिलेले वाक्य जर बरोबर असेल तर T लिहिलेला बॉक्स निवडा.
दिलेले वाक्य जर चुकीचे असेल तर F लिहिलेला बॉक्स निवडा.

1. The average of the squares of the first four prime numbers is 9.75.

पहिल्या चार मूळ संख्यांच्या वर्गांची सरासरी 9.75 आहे.

Answer: False

Solution:

Sum of the squares of the first 4 prime numbers = $4 + 9 + 25 + 49 = 87$.

So, the average = $87/4 = 21.75$

2. 209^2 is the product of two distinct three-digit numbers whose difference is 240.

209^2 ही संख्या अशा दोन वेगळ्या तीन अंकी संख्यांचा गुणाकार आहे, की ज्यांच्यातील फरक 240 आहे.

Answer: True

Solution:

$209^2 = 11 \times 11 \times 19 \times 19 = 121 \times 361$. So, $361 - 121 = 240$.

3. If $a_1 + a_2 + a_3 = 1$, $a_2 + a_3 + a_4 = 2$, $a_3 + a_4 + a_5 = 3$, , $a_{49} + a_{50} + a_1 = 49$
and $a_{50} + a_1 + a_2 = 50$; then, $a_1 + a_2 + a_3 + \dots + a_{50} = 425$.

जर $a_1 + a_2 + a_3 = 1$, $a_2 + a_3 + a_4 = 2$, $a_3 + a_4 + a_5 = 3$, , $a_{49} + a_{50} + a_1 = 49$

आणि $a_{50} + a_1 + a_2 = 50$; तर $a_1 + a_2 + a_3 + \dots + a_{50} = 425$.

Answer: True

Solution:

$$a_1 + a_2 + a_3 = 1$$

$$a_2 + a_3 + a_4 = 2$$

$$a_3 + a_4 + a_5 = 3$$

.

.

.

$$a_{49} + a_{50} + a_1 = 49$$

$$a_{50} + a_1 + a_2 = 50$$

Adding these, we get:

$$3(a_1 + a_2 + a_3 + \dots + a_{50}) = 1 + 2 + 3 + \dots + 50 = (50 \times 51)/2 = 1275$$

$$\text{So, } (a_1 + a_2 + a_3 + \dots + a_{50}) = 1275/3 = 425.$$

4. Product of two binomials $(3x + 2)$ and $(x - 7)$ results in the polynomial $(3x^2 + 19x - 14)$.

$(3x + 2)$ आणि $(x - 7)$ ह्या द्विपदींचा गुणाकार केल्यास $(3x^2 + 19x - 14)$ ही बहुपदी मिळते.

Answer: False

$$\text{Solution: } (3x + 2)(x - 7) = 3x(x - 7) + 2(x - 7)$$

$$= 3x^2 - 21x + 2x - 14$$

$$= 3x^2 - 19x - 14$$

5. The decimal representation of the fraction $\frac{5}{121}$ will terminate after a finite number of non-repeating digits.

$\frac{5}{121}$ हया अपूर्णाकाचे दशांश रूप काही ठराविक अनावर्ती अंकांनंतर समाप्त होईल.

Answer: False

Solution: It is a rational number in the p/q form, where p and q are both integers and q is not 0. Since the prime factor 11 is present in the denominator ($121 = 11 \times 11$), and it is neither 2 nor 5, the decimal representation will be a repeating decimal that goes on forever, not a terminating one.

6. You have four identical cubes with a side of 3 cm each.
If you stack them one on top of the other to create a single column (a tall cuboid), the final solid has a surface area that is exactly 50% greater than the surface area of a single 3 cm cube.

तुमच्याकडे चार एकसारखे प्रत्येक बाजू 3 सेमी असणारे घनाकृती ठोकळे आहेत.
जर तुम्ही त्यांना एकावर एक रचून एक उभा खांब (एक उंच इष्टिकाचिती) तयार केला,
तर त्या अंतिम आकृतीचे पृष्ठफळ हे एका 3 सेमी घनाकृतीच्या पृष्ठफळापेक्षा
50% ने अधिक असेल.

Answer: False

Solution:

For a 3 cm cube, $SA = 54$ square cm.

For the tall cuboid, dimensions (in cm) are $L = 3$, $W = 3$ and $H = 12$.

Total $SA = 2(LW + WH + LH) = 2(9 + 36 + 36) = 2 \times 81 = 162$ cubic cm.

Then, the increase $162 - 54 = 108$ is $(108/54) \times 100 = 200\%$ and not 50%.



7. Vertically opposite angles are equal because the linear pairs forming the vertically opposite angles are supplementary.

विरुद्ध कोन समान असतात कारण विरुद्ध कोन तयार करणारे रेषीय कोन पूरक असतात.

Answer: True

Solution:

When two lines intersect, they form pairs of vertically opposite angles.

Each vertically opposite angle is adjacent to a linear pair.

Since angles in a linear pair are supplementary, and each vertically opposite angle is supplementary to the same adjacent angle, the two vertically opposite angles must be equal. Thus, vertically opposite angles are equal because the linear pairs forming them are supplementary.

8. Every pentagon has exactly five diagonals.

प्रत्येक पंचकोनाला पाच कर्ण असतात.

Answer: True

Solution:

A pentagon has five vertices.

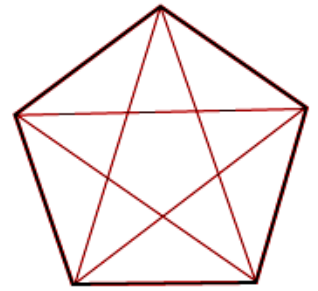
From any one vertex, you can draw diagonals to two non-adjacent vertices (you cannot draw a diagonal to itself or to its two neighbouring vertices). So, each vertex gives 2 diagonals.

That seems to give $5 \times 2 = 10$ diagonals.

However, each diagonal is counted twice (once from each end).

Therefore, the actual number of distinct diagonals is:

$$10/2 = 5$$



9. A triangle has side lengths 5, 12, and 13 cm.
A rectangle is constructed such that its area is equal to this triangle's area.
Then, the rectangle must have dimensions 10 cm x 6 cm.

एका त्रिकोणाच्या बाजूंची लांबी 5, 12 आणि 13 सेमी आहे.

एक आयत अशा प्रकारे तयार केला आहे की त्याचे क्षेत्रफळ ह्या त्रिकोणाच्या क्षेत्रफळाएवढे आहे.

तर, त्या आयताचे परिमाण 10 सेमी x 6 सेमी असणे आवश्यक आहे.

Answer: False.

Solution:

A triangle with side lengths 5, 12, and 13 cm has to be a right triangle with a base of 5 cm and a height of 12 cm. So, the area of the triangle = $\frac{1}{2} \times 5 \times 12 = 30$ sq. cm

But the area of the rectangle with dimensions 10 cm x 6 cm = 60 sq. cm \neq 30 sq. cm

10. If $x = \sqrt{7}$ and $y = \sqrt{28}$, then $(x + y)^2 = 35$

जर $x = \sqrt{7}$ आणि $y = \sqrt{28}$ असेल, तर $(x + y)^2 = 35$ होईल.

Answer: **False**

Solution:

$$\begin{aligned}\text{If } x = \sqrt{7} \text{ and } y = \sqrt{28}, \text{ then } (x + y)^2 &= x^2 + y^2 + 2xy \\ &= 7 + 28 + 2 (\sqrt{7}) (\sqrt{28}) \\ &= 35 + 2 (\sqrt{7}) (\sqrt{4}) (\sqrt{7}) \\ &= 35 + 2 (7) (2) \\ &= 35 + 28 \\ &= 63\end{aligned}$$

Section II : Each question carries 2 marks

विभाग II : प्रत्येक प्रश्नाला 2 गुण आहेत

For Q.11 to 20: Choose the correct option.

प्रश्न क्रमांक 11 ते 20: योग्य पर्याय निवडा.

11. In how many distinct ways can the letters of the word VARIETY be arranged so that the arrangement begins with R and ends with T?

"VARIETY" या शब्दातील अक्षरांची मांडणी अशा किती विशिष्ट प्रकारे करता येईल, जेणेकरून त्या मांडणीची सुरुवात 'R' अक्षराने होईल आणि शेवट 'T' अक्षराने होईल?

- a. 60 b. 120 c. 240 d. 720

Answer: b. 120

Solution:

The word VARIETY has 7 letters. Out of them, the positions of R and T are fixed. So, the remaining 5 letters can be arranged in $5 \times 4 \times 3 \times 2 \times 1 = 120$ ways.

12. $1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4}}} = \dots\dots\dots$

- a. $\frac{13}{30}$ b. $\frac{43}{30}$ c. $\frac{25}{4}$ d. $\frac{30}{43}$

Answer: b. $\frac{43}{30}$

Solution:

Start with the innermost denominator.

Step I:

$$3 + \frac{1}{4} = \frac{13}{4}$$

Step II:

$$\frac{1}{\frac{13}{4}} = \frac{4}{13}$$

Step III:

$$2 + \frac{4}{13} = \frac{30}{13}$$

Step IV:

$$\frac{1}{\frac{30}{13}} = \frac{13}{30}$$

Step V:

$$1 + \frac{13}{30} = \frac{43}{30}$$

13. Two contestants participated in an election with 100% turnout. The winner was elected by a margin of $\frac{1}{12}$ th of the total votes. If 150 of the winner's supporters had switched their votes to the opponent, the winner would have lost by a margin of $\frac{1}{6}$ th of the total votes. Find the total number of votes cast.

एका निवडणुकीत दोन उमेदवार होते आणि 100% मतदान झाले.

जिंकणारा उमेदवार एकूण मतांच्या $\frac{1}{12}$ इतक्या मतांच्या फरकाने निवडून आला.

जर जिंकणाऱ्या उमेदवाराच्या 150 समर्थकांनी आपली मते प्रतिस्पर्ध्याला दिली असती, म्हणजेच आपली मते बदलली असती, तर तो उमेदवार एकूण मतांच्या $\frac{1}{6}$ इतक्या फरकाने हरला असता. तर एकूण किती मते दिली गेली?

- a. 300 b. 600 c. 900 d. 1200

Answer: d. 1200

Solution:

METHOD I

Let A and B be the contestants. Suppose A gets 'a' votes and B gets 'b' votes.

Let T be the total number of votes cast.

Then, $a - b = T/12$ and $(b + 150) - (a - 150) = T/6$

i.e. $b - a + 300 = T/6$

Adding these, we get:

$$(a - b) + (b - a + 300) = T/12 + T/6$$

$$300 = T/12 + T/6$$

$$300 = 3T/12$$

$$300 = T/4$$

$$T = 300 \times 4 = 1200$$

Hence, the total number of votes cast is 1200.

OR

METHOD II

Let A and B be the contestants. Suppose A gets 'a' votes and B gets 'b' votes.

Let M be the margin with which A wins.

$$\text{Then, } a - b = M \text{ and } (b + 150) - (a - 150) = 2M \quad \dots \text{ As } T/6 = 2 \times T/12$$

Adding these, we get:

$$a - b + (b + 150) - (a - 150) = 3M$$

$$300 = 3M$$

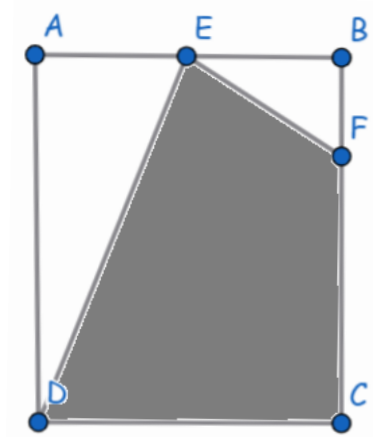
$$M = 100$$

Given that, A wins by a margin of $1/12$ of the total votes.

$$1/12 \times \text{Total votes} = 100$$

$$\text{Total votes} = 100 \times 12 = 1200$$

14. In the given rectangle ABCD, BF = 2 and FC = CD = 6. E is NOT the mid-point of AB. If the area of the shaded region is $3/4$ of the total area of the rectangle, find BE.



दिलेल्या आयत ABCD मध्ये, $BF = 2$ आणि $FC = CD = 6$ आहे.
E हा बाजू AB चा मध्यबिंदू नाही.
जर छायांकित भागाचे क्षेत्रफळ आयताच्या एकूण क्षेत्रफळाच्या $3/4$ पट असेल, तर BE ची किंमत काढा.

- | | |
|------|------|
| a. 2 | b. 4 |
| c. 6 | d. 8 |

Answer: b. 4

Solution:

Let BE be x .

$$\begin{aligned} \text{Area of the rectangle ABCD} &= CD \times BC = CD \times (BF + FC) \\ &= 6 \times (2 + 6) = 6 \times 8 = 48 \text{ sq units.} \end{aligned}$$

$$\text{Area of the shaded region CDEF} = 3/4 \times 48 = 36 \text{ sq units.}$$

Area of the non-shaded region = $48 - 36 = 12$ sq units. ... (i)

$$\begin{aligned}\text{But the area of the non-shaded region} &= \text{Area } \triangle ADE + \text{Area } \triangle BEF \\ &= \left(\frac{1}{2} \times AE \times AD\right) + \left(\frac{1}{2} \times BF \times BE\right) \\ &= \left(\frac{1}{2} \times (AB - BE) \times AD\right) + \left(\frac{1}{2} \times BF \times BE\right) \\ &= \left\{\frac{1}{2} \times (6 - x) \times 8\right\} + \left(\frac{1}{2} \times 2 \times x\right) \\ &= 4(6 - x) + x \\ &= 24 - 4x + x \\ &= 24 - 3x\end{aligned}$$

$$\begin{aligned}\text{But from (i), } 24 - 3x &= 12 \\ 24 - 12 &= 3x \\ 3x &= 12 \\ x &= 4\end{aligned}$$

15. If the fourth power of 3 is 81, what is the square root of 0.0081?

जर 3 च्या चौथ्या घाताची किंमत 81 असेल, तर 0.0081 या संख्येचे वर्गमूळ किती असेल?

- a. 0.009 b. 0.9 c. 0.09 d. 0.0009

Answer: c. 0.09

Solution:

$$3^4 = 9^2 = 81$$

We need the square root of 0.0081.

Decimal Rule: The number of decimal places in the square root is half the number of decimal places in the original number.

As 0.0081 has four decimal places, its square root must have $4 / 2 = 2$ decimal places.

As the square root of 81 is 9, the square root of 0.0081 is 0.09.

16. $\frac{\left(125^{\frac{n+2}{3}}\right)(5^{3n+2})}{(25^{n+1})(5^{2n})} = \dots\dots\dots$

- a. 0 b. 1 c. 5 d. 25

Answer: d. 25

Solution:

$$\frac{\left(125^{\frac{n+2}{3}}\right)(5^{3n+2})}{(25^{n+1})(5^{2n})} = \frac{\left(5^3\right)^{\frac{n+2}{3}}(5^{3n})(5^2)}{(5^{2n})(5^2)(5^{2n})} = \frac{(5^{n+2})(5^{3n})(5^2)}{(5^{2n})(5^2)(5^{2n})} = \frac{(5^n)(5^2)(5^{3n})(5^2)}{(5^{2n})(5^2)(5^{2n})} = 5^2 = 25$$

17. What should be added to $x^2 + 7x + 5$ so that the new polynomial is equal to the product of $(x + 4)$ and $(x + 3)$?

$x^2 + 7x + 5$ या बहुपदीत किती मिळवले असता येणारी नवीन बहुपदी ही $(x + 4)$ आणि $(x + 3)$ यांच्या गुणाकाराइतकी असेल?

- a. 6 b. -6 c. 7 d. -7

Answer: c. 7

Solution:

We have $(x + 4)(x + 3) = x^2 + 7x + 12$.

So, 7 should be added to the given polynomial $x^2 + 7x + 5$ to obtain $x^2 + 7x + 12$.

18. How many digits does the product $4^3 \times 5^8 \times 6$ have?

$4^3 \times 5^8 \times 6$ ह्या गुणाकाराचे उत्तर ही किती अंकी संख्या आहे?

- a. 10 b. 9 c. 8 d. 7

Answer: b. 9

Solution:

$$\begin{aligned} 4^3 \times 5^8 \times 6 &= 2^6 \times 5^8 \times 2 \times 3 \\ &= 2^7 \times 5^8 \times 3 \\ &= 2^7 \times 5^7 \times 5 \times 3 \end{aligned}$$

$$\begin{aligned}
&= (2 \times 5)^7 \times 5 \times 3 \\
&= 10^7 \times 15 \\
&= 150000000
\end{aligned}$$

Hence, there are 9 digits in all.

19. A library contains different types of books.

The ratio of biography books, science fiction books, and novels is 5 : 7 : 8.

In a particular month, additional books of each type are added to the library.

Then, the number of biography books increases by 40%, science fiction books by 50%, and novels by 75%. What is the new ratio of biography books, science fiction books, and novels in the library?

एका वाचनालयात विविध प्रकारची पुस्तके आहेत.

चरित्रात्मक, विज्ञान कथा आणि कादंबऱ्या ह्या प्रकारच्या पुस्तकांचे गुणोत्तर 5 : 7 : 8 असे आहे.

एका विशिष्ट महिन्यात प्रत्येक प्रकारची काही अतिरिक्त पुस्तके वाचनालयात आणली गेली.

त्यानंतर चरित्रात्मक पुस्तकांची संख्या 40% ने, विज्ञान कथांच्या पुस्तकांची संख्या 50% ने, आणि कादंबऱ्यांची संख्या 75% ने वाढते. तर आता त्या वाचनालयातील चरित्रात्मक, विज्ञान कथा आणि कादंबरी प्रकारातील

पुस्तकांचे नवीन गुणोत्तर काय असेल?

- | | |
|--------------|--------------|
| a. 2 : 3 : 4 | b. 6 : 7 : 8 |
| c. 6 : 8 : 9 | d. 1 : 2 : 3 |

Answer: a. 2 : 3 : 4

Solution:

Let the original numbers of each type be 5t, 7t and 8t, respectively.

They increase by 40%, 50% and 75% respectively.

So, new numbers are

$$(1.4 \times 5t) = 7t,$$

$$(1.5 \times 7t) = (10.5)t$$

$$(1.75 \times 8t) = 14t$$

$$\text{So, the new ratio} = 7 : (10.5) : 14 = 2 : 3 : 4$$

20. A shopkeeper sells two identical items for ₹ 3000 each.

On the first item, he makes a profit of 25%.

On the second item, he incurs a loss of 25%.

What is the net gain or loss percentage for the entire transaction?

एक दुकानदार प्रत्येकी ₹ 3000 रुपयांना दोन वस्तू विकतो.

पहिल्या वस्तूवर त्याला 25% नफा होतो. दुसऱ्या वस्तूवर त्याला 25% तोटा होतो.

या संपूर्ण व्यवहारात त्याला एकूण किती टक्के नफा किंवा तोटा झाला?

- | | | | |
|----|-------------------------|----|------------|
| a. | 0% (No Profit, No Loss) | b. | 6.25% Loss |
| | 0% (ना नफा, ना तोटा) | | 6.25% तोटा |
| c. | 5.75% Gain | d. | 100% Loss |
| | 5.75% नफा | | 100% तोटा |

Answer: **b. 6.25% Loss**

Solution:

The net result is always a loss when two items are sold for the same price, and the profit percentage on one equals the loss percentage on the other.

The loss is calculated on the lower Cost Price (CP) item, and the profit is calculated on the higher CP item, leading to an overall loss.

Section III : Each question carries 3 marks

विभाग III : प्रत्येक प्रश्नाला 3 गुण आहेत.

For Q.21 to 30: Write the correct two-digit natural number as an answer for each question.

प्रश्न क्रमांक 21 ते 30: उत्तर म्हणून योग्य दोन अंकी नैसर्गिक संख्या लिहा.

21. Find the total number of distinct points where the diagonals of a regular hexagon intersect.

सुसम षटकोनातील सर्व कर्णांच्या छेदनबिंदूंची संख्या शोधा.

Answer: 13

Solution:

To find the total number of distinct points where the diagonals of a regular hexagon cross inside, we can just count them by grouping the types of intersection points. A regular hexagon has 6 vertices and 9 diagonals.

We can categorize the intersection points into three distinct types:

1. The Center Point

What forms it: The three Long Diagonals (the ones connecting opposite corners) all cross right at the center.

Count: This is just 1 point.

2. The Inner Hexagon Points

What forms it: These points are created by the crossing of two Short Diagonals (diagonals that skip only one corner).

For example, the diagonal from corner 1 to corner 3 crosses the diagonal from corner 2 to corner 4.

Count: Due to the hexagon's symmetry, these crossings form a smaller, inner hexagon, giving us 6 points.

3. The Mid-Radius Points

What forms it: These points are created by a Long Diagonal crossing a Short Diagonal (where the diagonals don't share a vertex).

Count: There are 6 points of this type, which form a star shape connecting the inner hexagon to the center.

Total Distinct Intersection Points = 1 + 6 + 6 = 13.

22. There are three bags in a storeroom: one contains 5 kg of wheat, another contains 6 kg of rice, and the third contains 4 kg of dal. The total cost of all three bags is ₹ 1100. The cost of two wheat bags and one rice bag is ₹ 940. If the rice bag and the dal bag are priced the same, then what is the cost (in ₹) of wheat per kilogram?

एका गोदामात धान्यांची एकूण तीन पोती आहेत. त्यातील एका पोत्यात 5 किलो गहू, दुसऱ्या पोत्यात 6 किलो

तांदूळ आणि तिसऱ्यात 4 किलो डाळ आहे. तीन पोत्यांची मिळून किंमत 1100 रुपये आहे, तर गव्हाची दोन पोती आणि तांदळाचे एक पोते ह्यांची मिळून किंमत 940 रुपये आहे.

जर एका तांदळाच्या आणि एका डाळीच्या पोत्याची किंमत सारखीच असेल, तर एक किलो गव्हाची किंमत किती रुपये ते शोधा.

Answer: 52

Solution:

We have

$$W + R + D = 1100$$

$$2W + R = 940$$

$$R = D$$

Then

$$W + 2R = 1100$$

$$2W + R = 940$$

$$4W + 2R = 1880$$

$$3W = 780$$

$$W = 260 \quad \dots \text{one bag is 5 kg}$$

Therefore, the cost of wheat per kilogram = $260 \div 5 = 52$.

23. How many days are in the $2^{10} 3^8 5^3 6^{(-3)}$ seconds?

$2^{10} 3^8 5^3 6^{(-3)}$ सेकंद म्हणजे किती दिवस?

Answer: 45

Solution:

$$\text{Given that } 2^{10} 3^8 5^3 6^{(-3)} = 2^{10} 3^8 5^3 \{(2 \times 3)^{(-3)}\} = 2^{10} 3^8 5^3 2^{(-3)} 3^{(-3)} = 2^7 3^5 5^3$$

So, 1 day = 24 hours = 24 x 60 minutes = 24 x 60 x 60 seconds

$$= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 2 \times 2 \times 3 \times 5 \times 2 \times 2$$

$$= 2^7 3^3 5^2$$

Thus, $2^7 3^5 5^3 = 2^7 3^3 5^2 \times (3 \times 3 \times 5) = 45 \text{ days.}$

24. A group of men completed a piece of work in 60 days.
If there had been 13 more men, the work would have been completed 10 days earlier.
How many men were originally engaged in the work?

काही पुरुषांच्या एका गटाने एक काम 60 दिवसांत पूर्ण केले.
जर त्या गटात 13 पुरुष अधिक असले असते, तर ते काम 10 दिवस लवकर पूर्ण झाले असते.
तर, सुरुवातीला कामावर किती पुरुष होते?

Answer: **65**

Solution:

Here, the total work is constant and is equal to the number of men multiplied by the number of days: $W = M \times D$.

Let M be the original number of men and d be the original number of days, which is 60 days.

The total work in the first scenario = $60 \times M$.

In the second scenario:

The number of men is $(M + 13)$.

The number of days is $60 - 10 = 50$ days.

The total work in the second scenario = $50 \times (M + 13)$.

Since the total work is the same in both cases, we have:

$$60M = 50(M + 13)$$

$$60M = 50M + 650$$

$$60M - 50M = 650$$

$$10M = 650$$

$$M = 65 \quad \dots \text{ No. of men originally engaged in the work}$$

25. A sum of money earns ₹720 as simple interest in two years.
When the same sum is invested at the same rate of interest for two years under compound interest, it earns ₹763.20. Find the rate of interest per annum.

एका रकमेवर दोन वर्षासाठी सरळ व्याजाने 720 रुपये मिळतात.
तीच रक्कम, त्याच व्याजदराने दोन वर्षासाठी चक्रवाढ व्याजाने गुंतवल्यास 763.20 रुपये मिळतात.
तर, प्रतिवर्ष व्याजदर किती रुपये ते शोधा.

Answer: 12

Solution:

Given that the SI for two years = ₹ 720

CI for two years = ₹ 763.20

Time = 2 years

Then:

SI for 1 year = $720 / 2 = ₹ 360$

Difference between CI and SI for 2 years = $763.20 - 720 = ₹ 43.20$

This is the simple interest earned on the first year's simple interest.

Therefore,

Difference = $SI \times R/100$... R is the rate of interest per annum.

Hence, $43.20 = 360 \times R/100$

$4320 = 360 \times R$

$R = 4320/360 = 12$

The rate of interest per annum is 12%.

26. Anagha is travelling to the airport to catch a flight to another city.
Her auto is moving at a speed of 40 km/h, and the airport is 20 km from her house.
Fifteen minutes after she leaves, her brother realises that she has forgotten her phone charger at home and decides to follow her on his bike.
At what speed, in km/h, should her brother travel so that he reaches the airport at exactly the same time as Anagha?

अनघा दुसऱ्या शहरात विमानाने जाण्यासाठी विमानतळाकडे प्रवास करत आहे.
तिची रिक्शा 40 किमी/तास वेगाने जात आहे आणि विमानतळ तिच्या घरापासून २० किमी दूर आहे.
ती घरातून निघाल्यानंतर पंधरा मिनिटांनी तिच्या भावाला लक्षात येते की ती घरी तिचा फोन चार्जर घरीच विसरली आहे. म्हणून चार्जर देण्यासाठी तो तिच्या मागे त्याच्या दुचाकीवरून जाण्याचा निर्णय घेतो.

तर, अनघाच्या भावाने किती वेगाने (किमी/तास) प्रवास करायला हवा, जेणेकरून तो अनघा बरोबर त्याच वेळी विमानतळावर पोहोचेल, हे काढा.

Answer: 80

Solution:

Average Speed = Total Distance/ Total Time

Anagha's travel time = Total Distance/ Speed = $20/40 = \frac{1}{2}$ hr

The brother starts 15 minutes after Anagha.

To reach the airport at the exact same time, he must complete the journey in total time Anagha takes minus his 15-minute delay.

15 minutes = $15/60$ hrs = $\frac{1}{4}$ hrs

Her brother's available time = Her time - Delay = $\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ hrs

His speed in km/hr = Total Distance (in km) / Total Time (in hr) = $20 / (\frac{1}{4}) = 20 \times 4 = 80$

27. Find the highest power of 3 that completely divides the number:

$54 \times 545 \times 5454 \times 54545 \times 545454 \times 5454545 \times 54545454$

$54 \times 545 \times 5454 \times 54545 \times 545454 \times 5454545 \times 54545454$

हया संख्येस निःशेष भागणारा 3 चा सर्वोच्च घात शोधा.

Answer: 13

Solution:

A number is divisible by 3 if and only if the sum of its digits is divisible by 3.

A number is divisible by 3^k if it can be divided by 3 a total of k times.

To find the highest power of a prime number p that divides a product, we add the highest powers of p that divide each factor.

For 54, power = 3 (3 x 3 x 3 = 27 divides 54)

For 545, power = 0 (545 is not divisible by 3)

Similarly, for 54545, 5454545; power = 0 (Both are not divisible by 3)

For 5454 and 54545454, power = 3. For 545454, power = 4.

Thus, required power = $3 + 0 + 3 + 0 + 4 + 0 + 3 = 13$

28. Arun put ₹ 1 in his Piggy bank on the first day. On the second day, he put ₹ 2. From the third day onwards, he started adding together the amounts deposited in the bank on all the previous days and deposited that total into his piggy bank. After the seventh day, how much money will be there in his piggy bank?

अरुणने पहिल्या दिवशी त्याच्या गुल्लक मध्ये 1 रुपया टाकला तर दुसऱ्या दिवशी 2 रुपये टाकले.

तिसऱ्या दिवसापासून त्याने त्या अगोदरच्या प्रत्येक दिवशी टाकलेल्या रुपयांच्या बेरजेइतके रुपये गुल्लक मध्ये टाकले. तर, सातव्या दिवसानंतर त्याच्या गुल्लक मध्ये किती रुपये जमा झाले असतील?

Answer: 96

Solution:

Observe that: Arun is essentially doubling the amount.

Day (n)	Deposit (Dn)	Total Amount (Tn)	Calculation for Dn and Tn
1	₹ 1	₹ 1	$T_1 = D_1 = 1$
2	₹ 2	₹ 3	$T_2 = T_1 + D_2 = 1 + 2 = 3$
3	₹ 3	₹ 6	$D_3 = D_1 + D_2 = 1 + 2 = 3$
			$T_3 = T_2 + D_3 = 3 + 3 = 6$
4	₹ 6	₹ 12	$D_4 = D_1 + D_2 + D_3 = 1 + 2 + 3 = 6$
			$T_4 = T_3 + D_4 = 6 + 6 = 12$
5	₹ 12	₹ 24	$D_5 = D_1 + \dots + D_4 = 1 + 2 + 3 + 6 = 12$
			$T_5 = T_4 + D_5 = 12 + 12 = 24$
6	₹ 24	₹ 48	$D_6 = D_1 + \dots + D_5 = 12 + 12 = 24$
			$T_6 = T_5 + D_6 = 24 + 24 = 48$
7	₹ 48	₹ 96	$D_7 = D_1 + \dots + D_6 = 24 + 24 = 48$

			$T7 = T6 + D7 = 48 + 48 = 96$
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29. The perimeter of a scalene triangle with integer side lengths is 24 cm. Determine how many such distinct triangles are possible.

पूर्णांक बाजू असलेले आणि 24 सेमी परिमिती असलेले किती वेगवेगळे विषमभुज त्रिकोण असू शकतात?

Answer: 13

Solution:

Let a, b, and c be the sides of the triangle.

Then, $a + b + c = 24$ and $a + b > c$.

So $c < 12$ and $a + b > 12$.

By counting, we get 13 such scalene triangles.

30. Each letter from A to F represents a different digit from 0 to 9. The three numbers ABC, BGE, and DEF represent products with the same value (i.e., $A \times B \times C = B \times G \times E = D \times E \times F$). What is the maximum possible value of the product AGF?

A ते F पर्यंतचे प्रत्येक अक्षर 0 ते 9 पर्यंतचे वेगवेगळे अंक दर्शवते.

ABC, BGE आणि DEF हे गुणाकार समान आहेत.

तर, AGF या गुणाकाराचे जास्तीत जास्त संभाव्य मूल्य किती असेल?

A		D
B	G	E
C		F

Answer: 96

Solution:

Let the common value be P.

So, $ABC = BGE = DEF = P$

Since we want a maximum and all digits are distinct, $B \neq 0$ (otherwise $P = 0$).

So, we can cancel B from the first two products: $AC = GE \quad \dots (I)$

So, A, C must be a factor pair of the same 1–digit-by-1–digit product as G, E, using four distinct digits.

Now we want to maximise the product AGF.

The best way to make that large is to make A and F as large as possible, but note:

F appears in the product DEF, so if F is large, then D x E must be relatively small.

From (I), if A is large, then G tends to be smaller (because A x C is at most $9 \times 8 = 72$ with distinct digits).

A workable high-product equality for (I) with distinct digits is: $6 \times 3 = 2 \times 9 = 18$

So take A = 6, C = 3, G = 2, E = 9 (all distinct).

Now pick B so that $P = A \times B \times C = 6 \times B \times 3 = 18 \times B$ can also be written as

$D \times E \times F = D \times 9 \times F$.

Choosing B = 4 gives $P = 18 \times 4 = 72$. Then we need: $D \times 9 \times F = 72 \Rightarrow D \times F = 8$

With unused digits, we can take D = 1, F = 8 (distinct from the others).

This produces:

$$A \times B \times C = 6 \times 4 \times 3 = 72$$

$$B \times G \times E = 4 \times 2 \times 9 = 72$$

$$D \times E \times F = 1 \times 9 \times 8 = 72$$

So, $A \times G \times F = 6 \times 2 \times 8 = 96$, and this is the maximum.

One optimal assignment is:

(A, B, C, D, E, F, G) = (6, 4, 3, 1, 9, 8, 2)

Another also exists:

A = 8, B = 9, C = 1, D = 3, E = 4, F = 6, G = 2, which again gives $A \times G \times F = 96$.
