

**Bhaskaracharya Pratishthana, Pune**

**Bhaskaracharya Mathematics Talent Search Competition (BMTSC)**

**AY 2025-2026**

**Round II**

Time: 3.00 pm to 5.30 pm

Max. Marks: 60

Saturday, 07/02/2026

**Instructions for the candidates:**

- The question paper contains 8 Questions of 4 Marks and 4 Questions of 7 Marks.
- All questions are compulsory. There is no internal option.
- Use of calculators is NOT allowed.
- Draw neat and labelled diagrams, wherever necessary.
- Give proper justification for all your answers.
- If you are using a trial-and-error approach, provide the results of all attempts.

**Section I: Each question carries 4 marks**

**विभाग I: प्रत्येक प्रश्नाला 4 गुण आहेत.**

1. A literary prize fund of ₹1,000 was exhausted exactly by purchasing 12 books of two different types: 'Higher Mathematics' and 'Basic Mathematics.'

If a 'Higher Mathematics' book costs ₹20 more than a 'Basic Mathematics' book, and a 'Basic Mathematics' book is priced at ₹70, determine the number of students who received the 'Higher Mathematics' book.

साहित्यविषयक पारितोषिकांसाठी राखून ठेवलेला 1000 रुपये निधी उच्च गणित आणि सुगम गणित अशा दोन प्रकारच्या एकूण 12 पुस्तकांसाठी उपयोगात आणला गेला.

जर एका उच्च गणिताच्या पुस्तकाची किंमत एका सुगम गणिताच्या पुस्तकाच्या किंमतीपेक्षा 20 रुपयांनी

जास्त असेल व एका सुगम गणिताच्या पुस्तकाची किंमत 70 रुपये असेल, तर उच्च गणिताचे पुस्तक पारितोषिक म्हणून मिळालेल्या मुलांची संख्या काढा.

**Answer:** 8

**Solution:**

Let the number of 'Higher Mathematics' books be  $x$ .

So, the number of 'Basic Mathematics' books will be  $(12 - x)$ .

Cost of Basic Mathematics book = ₹ 70 and that of the Higher Mathematics book = ₹ 90

$$\text{So, } 90x + 70(12 - x) = 1000$$

$$90x + 840 - 70x = 1000$$

$$20x = 160$$

$$x = 8$$

So, 8 students received 'Higher Mathematics' books of Rs. 90 each.

2. If  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$  and  $abc = 2$ , find the value of  $a^2b^2c + ab^2c^2 + a^2bc^2$ .

जर  $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$  आणि  $abc = 2$ , तर  $a^2b^2c + ab^2c^2 + a^2bc^2$  ची किंमत काढा.

**Answer:** 4

**Solution:**

$$a^2b^2c + ab^2c^2 + a^2bc^2 = abc(ab + bc + ac)$$

Multiply and divide by  $abc$  to get:

$$\begin{aligned}
&= \frac{(abc)^2 (ab + bc + ac)}{(abc)} \\
&= (abc)^2 \left( \frac{ab}{abc} + \frac{bc}{abc} + \frac{ac}{abc} \right) \\
&= (abc)^2 \left( \frac{1}{c} + \frac{1}{a} + \frac{1}{b} \right) \\
&= 2^2 (1) \\
&= 4
\end{aligned}$$

3. Two labourers, Aju and Biju, are assigned two identical projects. They begin work simultaneously on their respective projects. After 4 days, they swap projects. Aju completes his new assignment in 10 additional days. Immediately upon finishing, he joins Biju to help complete the remaining portion of the second project. If they finish the second project together in exactly 2 days, determine the ratio of Aju's work rate to Biju's work rate.

अजू आणि बिजू या दोन मजुरांना दोन एकसारखी कामे देण्यात आली. त्यांनी आपापल्या कामाला एकाच वेळी सुरुवात केली. चार दिवसांनंतर, त्यांनी आपल्या कामांची आपापसात अदलाबदल केली. अजूने त्याचे नवीन मिळालेले काम पुढील दहा दिवसांत पूर्ण केले. ते काम संपवल्यावर लगेचच तो बिजूला दुसऱ्या कामाचा उर्वरित भाग पूर्ण करण्यास मदत करण्यासाठी सामील झाला. जर त्यांनी मिळून दुसरे काम बरोबर दोन दिवसांत पूर्ण केले असेल, तर अजूच्या कार्यक्षमतेचे (काम करण्याच्या वेगाचे) बिजूच्या कार्यक्षमतेशी असणारे गुणोत्तर काढा.

**Answer:**      **2 : 1**

**Solution:**

Let the work done by Aju in 1 day be  $W_1$ , and that by Biju in 1 day be  $W_2$ .  
Since both work projects are identical, the work done on both is the same.

Work done on project I =  $4W_2 + 10W_1$

Work done on project II =  $4W_1 + 10W_2 + 2(W_1 + W_2)$

$$= 4W_1 + 10W_2 + 2W_1 + 2W_2$$

$$= 6W_1 + 12W_2$$

Therefore,  $4W_2 + 10W_1 = 6W_1 + 12W_2$

$$4W_1 = 8W_2$$

$$W_1/W_2 = 8/4$$

$$W_1/W_2 = 2/1$$

Thus, the ratio of Aju's work speed to Biju's work speed is 2 : 1.

4. Find three natural numbers such that the second number is one more than the third number, the product of the first and second number equals 3,48,750 and the product of the first and third number equals 3,47,200.

अशा तीन नैसर्गिक संख्या शोधा ज्यातील दुसरी संख्या तिसऱ्या संख्येपेक्षा एक ने अधिक आहे, पहिल्या व दुसऱ्या संख्येचा गुणाकार 3,48,750 आहे आणि पहिल्या व तिसऱ्या संख्येचा गुणाकार 3,47,200 आहे.

**Answer:** 1550, 225 and 224

**Solution:**

Let the three natural numbers be a, b, and c such that:

$$b = c + 1, ab = 348750 \text{ and } ac = 347200.$$

$$\text{As } b = c + 1, ab = a(c + 1) = ac + a$$

$$\text{So, } ab - ac = 348750 - 347200 = 1550 = ac + a - ac = a$$

$$\text{So, } a = 1550; b = 348750 \div 1550 = 34875 \div 155 = 6975 \div 31 = 225 \text{ and } c = 225 - 1 = 224$$

$$\text{So, } a = 1550, b = 225 \text{ and } c = 224$$

5. An amount of ₹10,000 is invested in a scheme at an annual interest rate of 10%. At the end of each year, the interest earned is added to the principal and reinvested at the same rate. This process continues for three years. Let 'I' be the total interest accumulated over this period. If we want to earn the same total interest 'I' over three years using a standard Simple Interest method at the same rate, what initial principal amount would need to be invested? (Round off your answer to two decimal places).

एका योजनेत ₹10,000 इतकी रक्कम 10% वार्षिक व्याजदराने गुंतवली जाते. प्रत्येक वर्षाच्या शेवटी, मिळणारे व्याज मुद्दलामध्ये मिळवले जाते आणि ती एकूण रक्कम पुन्हा त्याच दराने परत गुंतवली जाते. ही प्रक्रिया तीन वर्षांपर्यंत चालू राहते. या कालावधीत जमा झालेल्या एकूण व्याजाला 'I' मानूया. जर आपल्याला तेवढेच एकूण 'I' इतके व्याज तीन वर्षांत सरळव्याज पद्धतीने आणि त्याच व्याजदराने मिळवायचे असेल, तर सुरुवातीला किती मुद्दल गुंतवावे लागेल? (तुमचे उत्तर दशांश चिन्हांनंतर दोन घरांपर्यंत लिहा).

**Answer:** ₹11033.33 or ₹ 11033  $\frac{1}{3}$

**Solution:**

**Step 1: Calculate the total interest I accumulated:**

Since the interest is added to the principal at the end of each year, we apply

SI formula  $I = (P \times R \times T)/100$  for each year individually.

**Year 1:** Principal = ₹10000, Interest =  $\{10000 \times 10 \times 1\} / \{100\} = ₹1000$

**Year 2:** Principal = 10000 + 1000 = ₹11000, Interest =  $\{11000 \times 10 \times 1\} / \{100\} = ₹1100$

**Year 3:** Principal = 11000 + 1100 = ₹12100, Interest =  $\{12100 \times 10 \times 1\} / \{100\} = ₹1210$

Total Interest ₹3310

**Step 2: Calculate the required Principal for Standard Simple Interest:**

We want to find a new principal that gives the same total interest  $I = ₹3310$  over  $T = 3$  years at  $R = 10\%$  using the standard simple interest formula.

$$3310 = (P \times 10 \times 3) / 100$$

$$331000 = P \times 30$$

$$P = 33100 / 3 = 11033.3333\dots$$

6. An expression is defined as the product  $(2^2 - 1)(3^2 - 1)(4^2 - 1) \dots (n^2 - 1)$ .

Then:

- (i) What is the smallest natural number  $n > 1$  for which this product is exactly a perfect square?
- (ii) What is the next natural number  $n$  after that, for which the product is again a perfect square?

$(2^2 - 1)(3^2 - 1)(4^2 - 1) \dots (n^2 - 1)$  हा एका पदावलीचा गुणाकार आहे.

तर -

- (i) एकपेक्षा मोठी असणारी परंतु लहानात लहान अशी नैसर्गिक संख्या  $n$  कोणती, ज्यासाठी वरील गुणाकार एक पूर्ण वर्ग संख्या असेल?
- (ii) त्यानंतर येणारी पुढची अशी नैसर्गिक संख्या  $n$  कोणती, ज्यासाठी वरील गुणाकार पुन्हा एक पूर्ण वर्ग संख्या असेल?

**Answer:** (i) 8 (ii) 49

**Solution:**

Each part of the product looks like  $k^2 - 1$ .

$$k^2 - 1 = (k - 1)(k + 1).$$

$$\begin{aligned} \text{Let } P &= [(2 - 1)(2 + 1)] [(3 - 1)(3 + 1)] [(4 - 1)(4 + 1)] \dots [(n - 1)(n + 1)] \\ &= (1 \times 3)(2 \times 4)(3 \times 5)(4 \times 6)(5 \times 7) \dots [(n - 1)(n + 1)] \end{aligned}$$

To make a product a perfect square, every number in its prime factorisation must appear an even number of times.

Looking at our list of numbers, let's see how many times each number appears:

The number 1 appears once (at the start).

The number 2 appears once (from the  $(3 - 1)$  part).

The number 3 appears twice (from  $(2 + 1)$  and  $(4 - 1)$ ).

The number 4 appears twice (from  $(3 + 1)$  and  $(5 - 1)$ ).

This pattern continues for every number up to  $(n - 1)$ .

At the end, the numbers  $n$  and  $(n + 1)$  appear only once.

So, the product can be written as:

$$P = (1 \times 2) (3^2 \times 4^2 \times 5^2 \dots (n - 1)^2) (n (n + 1))$$

Since all the numbers from 3 to  $(n - 1)$  are already squared, they are already "perfect."

For the whole product to be a perfect square, the remaining "unpaired" numbers must multiply to a square.

The unpaired numbers are:  $(1 \times 2 \times n \times (n + 1))$

So, we need  $(2 \times n \times (n + 1))$  to be a perfect square.

Since  $n$  and  $(n + 1)$  are consecutive numbers (they have no common factors), there are only two ways this can happen:

- $n$  is a square, and  $(n + 1)$  is 2 times a square.
- $n$  is 2 times a square, and  $(n + 1)$  is a square.

(i) Test small values of  $n$  using the two rules.

Try  $n = 2$ :  $2 \times 2 \times 3 = 12$  (Not a square)

Try  $n = 3$ :  $2 \times 3 \times 4 = 24$  (Not a square)

Try  $n = 8$ :

Here  $n = 8 = 2 (2^2)$  and  $n + 1 = 9 = 3^2$  (a perfect square)

Check:  $2n (n + 1) = 2 \times 8 \times 9 = 144$  (a perfect square)

Hence, the smallest such whole number is 8.

(ii) Look for the next  $n$  where either  $n$  or  $n + 1$  is a square, and the other is 2 times a square.

We already found  $n = 8$  (where  $n + 1 = 9$ ).

Let's check the next few squares for  $n$  or  $n + 1$ :

For  $n = 49$ :

$$n = 49 = 7^2$$

(a perfect square)

$$n + 1 = 50 = 2 \times 25 = 2(5^2)$$

$$\text{Check: } 2n(n + 1) = 2 \times 49 \times 50 = 4900 = 70^2$$

(a perfect square)

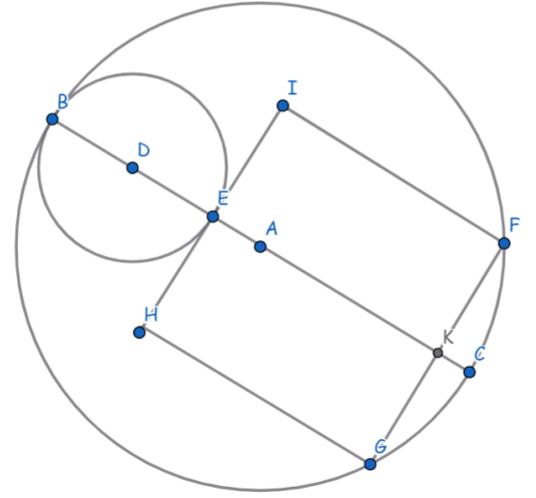
Thus, the next such whole number  $n$  is 49.

7. In the figure, A and D are the centres of the bigger and smaller circles, respectively.

The smaller circle touches the square IHGF at the midpoint of side IH. The radius of the bigger circle is 30 cm, and the side of the square IHGF is 36 cm.

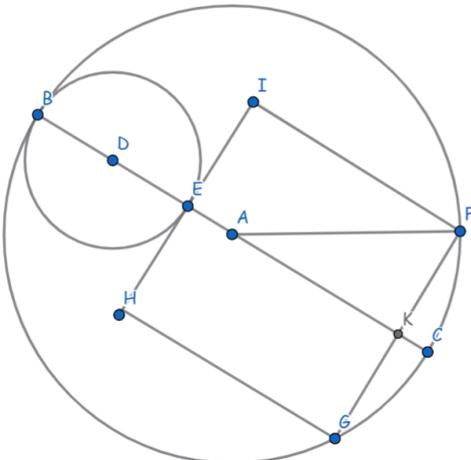
Find the radius of the smaller circle in cm.

दिलेल्या आकृतीमध्ये A आणि D हे अनुक्रमे मोठ्या आणि लहान वर्तुळांचे केंद्रबिंदू आहेत. IHGF ह्या चौरसाला लहान वर्तुळ जिथे स्पर्श करते तो बाजू IH चा मध्यबिंदू आहे. जर मोठ्या वर्तुळाची त्रिज्या 30 सेमी असेल आणि IHGF या चौरसाची बाजू 36 सेमी असेल, तर लहान वर्तुळाची त्रिज्या किती सेमी हे शोधा.



**Answer:** 9 cm

**Solution:**



Given:

$$IH = HG = GF = FI = 36 \text{ cm}$$

$$EH = EI = 18 \text{ cm}$$

$$AB = AC = 30 \text{ cm}$$

circle

... IHGF is a square.

... E is mid-point of IH

... radii of bigger

To find:

BD, the radius of the smaller circle.

Construction:

Join AF = 30 cm. ... radius of bigger circle

Then,  $\triangle AKF$  becomes a right-angled triangle with base

AK, height KF and hypotenuse AF.

Also, KF =  $\frac{1}{2}$  of GF = 18 cm ... symmetry of IHGF

So,  $AF^2 = AK^2 + KF^2$  ... Pythagorean Theorem in  $\triangle AKF$

$$30^2 = AK^2 + 18^2$$

$$AK^2 = 30^2 - 18^2 = 900 - 324 = 576$$

$$AK = 24 \text{ cm.}$$

Now, EK = EA + AK

$$36 = EA + 24$$

... EK = HG by symmetry

$$EA = 12$$

Thus, BE = AB - AE = 30 - 12 = 18 cm

But BE is the diameter of the smaller circle.

Hence, the required radius BD =  $\frac{1}{2}$  of BE =  $\frac{1}{2} \times 18 = 9$  cm.

8. The diagram below displays three card towers of varying heights:

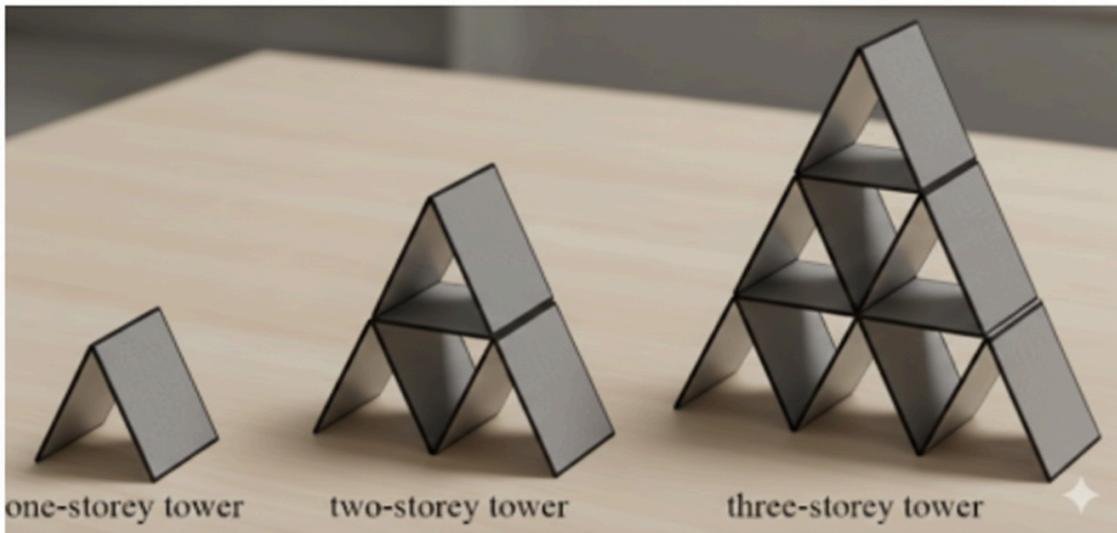
A one-storey tower requires 2 cards, a two-storey tower requires 7, and a three-storey tower requires 15 cards. Based on this progression, what is the maximum number of storeys possible if you have a total of 2026 cards?

खालील चित्रात पत्यांपासून तयार केलेले वेगवेगळ्या उंचीचे तीन मनोरे दर्शवण्यात आले आहेत.

एक मजली मनोरा तयार करण्यासाठी 2 पत्ते लागतात, दोन मजली मनोऱ्यासाठी 7 पत्ते लागतात

आणि तीन मजली मनोऱ्यासाठी 15 पत्ते लागतात. या रचनेचा विचार करता, तुमच्याकडे जर एकूण

2026 पत्ते असतील, तर त्यातून जास्तीत जास्त किती मजल्यांचा मनोरा तयार करता येईल?



**Answer:** 36

**Solution:**

Let's see how many new cards we add every time we add a storey at the bottom:

1 Storey: 2 cards

2 Storeys: 7 cards (We added 5 cards to the first storey)

3 Storeys: 15 cards (We added 8 cards to the two-storey tower)

So, the numbers we are adding are: 5, 8, 11, ...

So, every time we add a new storey at the bottom, we need 3 more cards than we did for the previous floor's addition.

Storeys (n)	New Cards Added	Total Cards
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1	2	2
2	$2 + 3 = 5$	$2 + 5 = 7$
3	$5 + 3 = 8$	$7 + 8 = 15$
4	$8 + 3 = 11$	$15 + 11 = 26$
5	$11 + 3 = 14$	$26 + 14 = 40$

For a tower with  $n$  storeys, the total cards =  $(3 \times (n \times n) + n) / 2$

Let's check it with 3 storeys.

For  $n = 3$ :

$$\text{Total cards} = (3 \times (3 \times 3) + 3) / 2 = ((3 \times 9) + 3) / 2 = (27 + 3) / 2 = 30/2 = 15 \text{ (Correct!)}$$

Since 2026 is a large number, let's try some "big" guesses for the number of storeys  $n$ :

For  $n = 30$ :

$$\text{Total cards} = (3 \times (30 \times 30) + 30) / 2 = ((3 \times 900) + 30) / 2 = (2700 + 30) / 2 = 1365$$

For  $n = 35$ :

$$\text{Total cards} = (3 \times (35 \times 35) + 35) / 2 = ((3 \times 1225) + 35) / 2 = (3675 + 35) / 2 = 1855 \text{ (closer)}$$

For  $n = 40$ :

$$\text{Total cards} = (3 \times (40 \times 40) + 40)/2 = ((3 \times 1600) + 40)/2 = (4800 + 40)/2 = 2420$$

Let's check 36 and 37 storeys:

For  $n = 36$ :

$$\text{Total cards} = (3 \times (36 \times 36) + 36)/2 = ((3 \times 1296) + 36)/2 = (3888 + 36)/2 = 1962$$

(This works! We have 2026 cards, so we have 64 cards left over).

For  $n = 37$ :

$$\text{Total cards} = (3 \times (37 \times 37) + 37)/2 = ((3 \times 1369) + 37)/2 = (4107 + 37)/2 = 2072$$

(This is too many; we only have 2026 cards).

Thus, at most 36 such storeys can be built using 2026 cards.

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**Section II: Each question carries 7 marks**

विभाग II: प्रत्येक प्रश्नाला 7 गुण आहेत.

9. Do as directed:

दिलेल्या सूचनांनुसार कृती करा.

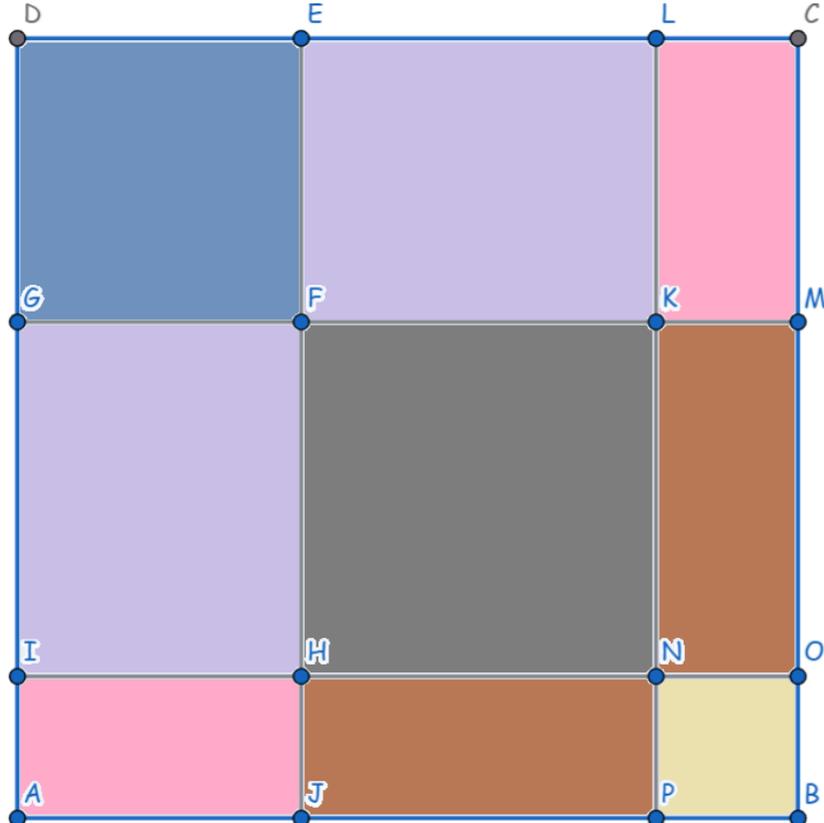
(i) Evaluate the following product and express the result in its most simplified form:

खालील पदावलीचा गुणाकार करा आणि संक्षिप्त रूपात उत्तर लिहा.

$$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$$

(ii) Determine the simplified expansion for the square of the trinomial  $(a + b + c)$ , using the image; where area of the square EDGF =  $a^2$ , area of the square KFHN =  $b^2$ , and area of the square ONPB =  $c^2$  for positive integers  $a, b, c$ .

दिलेल्या चित्राच्या आधारे  $(a + b + c)$  ह्या त्रिपदीचा विस्तार करा आणि संक्षिप्त रूपात उत्तर लिहा. येथे  $a, b, c$  हे धन पूर्णांक असून, चौरस EDGF चे क्षेत्रफळ  $a^2$ , चौरस KFHN चे क्षेत्रफळ  $b^2$  आणि चौरस ONPB चे क्षेत्रफळ  $c^2$  आहे.



Now, using the results obtained in (i) and (ii) above, solve the following:

आता, वरील (i) आणि (ii) मध्ये मिळालेल्या निष्कर्षांच्या आधारे, खालील प्रश्न सोडवा:

- (iii) Evaluate the expression  $x^3 + y^3 - 15xy + 125$  given the condition that  $x + y = -5$ .  
जर  $x + y = -5$  असेल, तर  $x^3 + y^3 - 15xy + 125$  या बैजिक राशीची किंमत काढा.
- (iv) There are three squares with sides having lengths of  $2p$ ,  $4q$  and  $3r$ , respectively. Find the difference in the sum of the areas of these three squares and the area of a square having the side as the sum of the sides of these three squares.

अनुक्रमे  $2p$ ,  $4q$  आणि  $3r$  बाजू असलेल्या तीन चौरसांचा विचार करा.

या तिन्ही चौरसांच्या क्षेत्रफळांची बेरीज आणि ज्याची बाजू या तिन्ही चौरसांच्या बाजूंच्या बेरजेइतकी आहे अशा एका मोठ्या चौरसाचे क्षेत्रफळ, यांमधील फरक शोधा.

**Answer:**

$$(i) \quad (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ac) = a^3 + b^3 + c^3 - 3abc$$

$$(ii) \quad (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(iii) \quad x^3 + y^3 - 15xy + 125 = 0$$

$$(iv) \quad (2p + 4q + 3r)^2 - [(2p)^2 + (4q)^2 + (3r)^2] = 16pq + 24qr + 12pr = 4(4pq + 6qr + 3pr)$$

**Solution:**

- (i) Multiplying each term in the first bracket by each term in the second bracket, we get,

$$\begin{aligned} & (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca) \\ &= a^3 + ab^2 + ac^2 - a^2b - abc - ca^2 + ba^2 + b^3 + bc^2 - ab^2 - b^2c - abc + ca^2 + cb^2 + c^3 \\ & \quad - abc - bc^2 - c^2a \\ &= a^3 + b^3 + c^3 - 3abc \end{aligned}$$

(All like terms highlighted in the same colours get cancelled).

$$(ii) \quad \text{Area of square EDGF} = a^2 \quad \text{So, side} = a$$

$$\text{Area of square KFHN} = b^2 \quad \text{So, side} = b$$

$$\text{Area of square ONPB} = c^2$$

$$\text{So, side} = c$$

Using these values of side lengths, we get the areas of the rectangles as follows:

$$\text{Area of Rect LEFK} = \text{Area of Rect FGIH} = ab$$

$$\text{Area of Rect CLKM} = \text{Area of Rect HIAJ} = ca$$

$$\text{Area of Rect MKNO} = \text{Area of Rect NHJP} = bc$$

$$\text{Area of the square of side } (a + b + c)$$

$$= \text{Sum of areas of three given squares} + \text{areas of the six rectangles listed above}$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$\text{Therefore, } (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

- (iii) Comparing  $x^3 + y^3 + 125 - 15xy$  to the identity obtained in (i)  $a^3 + b^3 + c^3 - 3abc$ , we can see that:

$$a = x, b = y \text{ and } c = 5$$

$$(\text{as } 3abc = 3 \cdot 5 \cdot x \cdot y = 15xy)$$

$$\text{So } x^3 + y^3 + 125 - 15xy = (x + y + 5)(x^2 + y^2 + 25 - xy - 5x - 5y)$$

$$\text{But it is given that } x + y = -5.$$

$$\text{Therefore } (x + y + 5) = 0$$

$$\text{So the value of the given expression} = 0.$$

- (iv) We need to find the difference between the expressions:

$$[(2p)^2 + (4q)^2 + (3r)^2] \text{ and } (2p + 4q + 3r)^2.$$

If we compare  $(2p + 4q + 3r)^2$  with  $(a + b + c)^2$  in (ii) and apply the result obtained in (ii), we get:

$$(2p + 4q + 3r)^2 = [(2p)^2 + (4q)^2 + (3r)^2] + 2(2p)(4q) + 2(4q)(3r) + 2(3r)(2p)$$

$$= [(2p)^2 + (4q)^2 + (3r)^2] + 16pq + 24qr + 12pr$$

$$\text{Therefore, } (2p + 4q + 3r)^2 - [(2p)^2 + (4q)^2 + (3r)^2] = 16pq + 24qr + 12pr$$

$$= 4(4pq + 6qr + 3pr)$$

10. A set of seven positive integers  $(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$  is arranged in the following order:  $x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6 \leq x_7$

This sequence has following constraints:

- Range: Each integer is between 1 and 26, inclusive.
- Mean: The average of the sequence is 20.
- Median: The middle value is 18.
- Mode: No other value appears as often as 25.

Find the minimum possible value for the first term  $x_1$ .

एका धन पूर्णांक संख्यांच्या संचामधील सर्व सात संख्या  $(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$  ह्या  $x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5 \leq x_6 \leq x_7$  अशा रीतीने मांडल्या आहेत.

ही संख्यांची क्रमिका खालील अटी पूर्ण करते:

- विस्तार: क्रमिकेतील पूर्णांक संख्या 1 ते 26 पैकी आहेत.
- मध्यमान: क्रमिकेतील संख्यांची सरासरी 20 आहे.
- मध्यक: क्रमिकेच्या बरोबर मध्यभागी 18 ही संख्या आहे.
- बहुलक: दुसरी कोणतीही संख्या 25 ही संख्या जितक्या वेळेस येते, तितक्या वेळेस येत नाही.

तर, क्रमिकेतील पहिले पद  $x_1$  ची लहानात लहान संभाव्य किंमत शोधा.

**Answer:** 12

**Solution:**

To find the minimum possible value for  $x_1$ , we need to make all the other numbers in the sequence as large as possible while staying within the given constraints.

Total sum =  $7 \times 20 = 140$ .

Median = Middle term = 4th term =  $x_4 = 18$ .

Mode = 25 appears the most; i.e., it's the unique mode.

Now, to make  $x_1$  as small as possible, we need the sum  $(x_2 + x_3 + x_4 + x_5 + x_6 + x_7)$  to be as large as possible.

Since the maximum value is 26, the largest possible values for the end of the sequence are 25 and 26.

To satisfy the "Unique Mode = 25" rule, 25 must appear at least twice.

If we use three 25s, we maximize the sum while keeping 25 as the mode.

However, we can use 26 as long as it doesn't appear more than or equal to the number of times 25 appears.

Let's set  $x_5 = 25$ ,  $x_6 = 25$ , and  $x_7 = 26$ .

(Mode 25 appears twice; 26 appears once).

Here, the sum is 76.

This is the highest possible sum for the last three terms while keeping 25 as the unique mode.

Now, we already know that  $x_4 = 18$ .

To keep  $x_1$  small,  $x_2$  and  $x_3$  must be as large as possible.

Because the sequence is non-decreasing, the largest  $x_2$  and  $x_3$  can be is the value of the median. Therefore,  $x_2 = x_3 = 18$ .

Now we add all the maximised terms and subtract from the total sum of 140.

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 140$$

$$x_1 + 18 + 18 + 18 + 25 + 25 + 26 = 140 \quad (\text{Making } x_1 = 10)$$

But in this specific set, 18 appears three times, making it the mode, which contradicts the rule that 25 is the unique mode. To fix this, we must reduce the frequency of 18s.

We must change  $x_2$  and  $x_3$  to be less than 18 so that 25 (appearing twice) becomes the unique mode. However, reducing  $x_2$  and  $x_3$  making  $x_1$  larger.

We must have  $x_3 = 17$  and  $x_2 = 16$ . This makes  $x_1 = 13$ .

To keep  $x_1$  the smallest, we instead increase the frequency of 25.

Let  $x_5 = 25$ ,  $x_6 = 25$ , and  $x_7 = 25$  (25 now appears three times).

Now, we can keep  $x_2 = 18$  and  $x_3 = 18$  because 18 also appears three times.

But the mode would not be "unique."

To make 25 the unique mode, we must ensure that 18 appears only twice.

We set  $x_3 = 18$  and  $x_2 = 17$  (this is the minimum value smaller than 18).

$$x_1 + 17 + 18 + 18 + 25 + 25 + 25 = 140$$

$$x_1 + 128 = 140$$

$$x_1 = 12 \quad \dots \text{ the smallest possible value}$$

(Sequence: 12, 17, 18, 18, 25, 25, 25)

11. Here are nine unique shapes. Each small square is of unit length.

**Task:**

To arrange these shapes into a 3 x 3 grid based on two specific geometric properties - area and perimeter.

**Rules of the arrangement:**

- Moving from left to right across any row, the area of the shapes must strictly increase.
- Moving from top to bottom down any column, the perimeter of the shapes must strictly increase.

- All the shapes placed in the middle column must have the same area.
- All the shapes placed in the middle row must have the same perimeter.

Explain the step-by-step systematic reasoning or mathematical strategy applied by you to arrange these shapes in the given 3 x 3 grid.

Also, determine their unique positions in the grid.

येथे नऊ वैशिष्ट्यपूर्ण आकार दिले आहेत. प्रत्येक लहान चौरस एक एकक लांबीचा आहे.

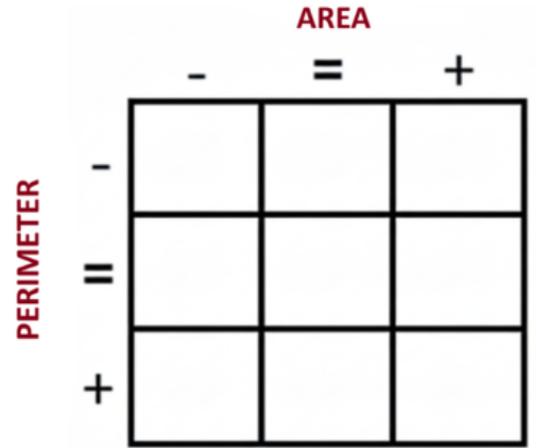
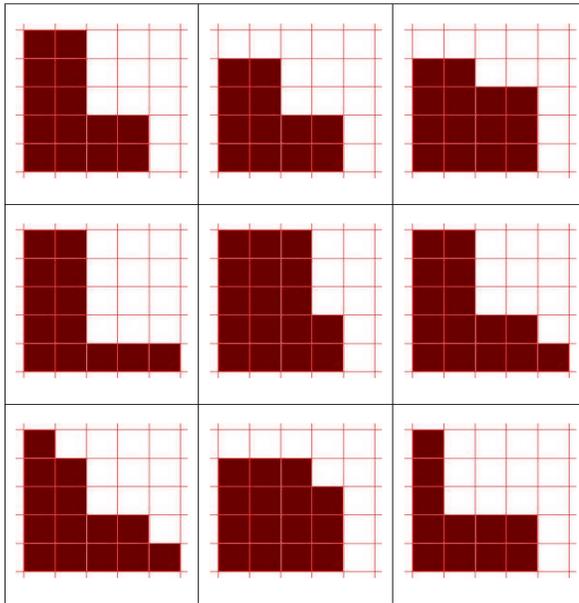
कार्य:

क्षेत्रफळ आणि परिमिती या दोन विशिष्ट भौमितिक गुणधर्मांच्या आधारे या आकारांची 3 x 3 च्या ग्रिड मध्ये मांडणी करणे.

मांडणीचे नियम:

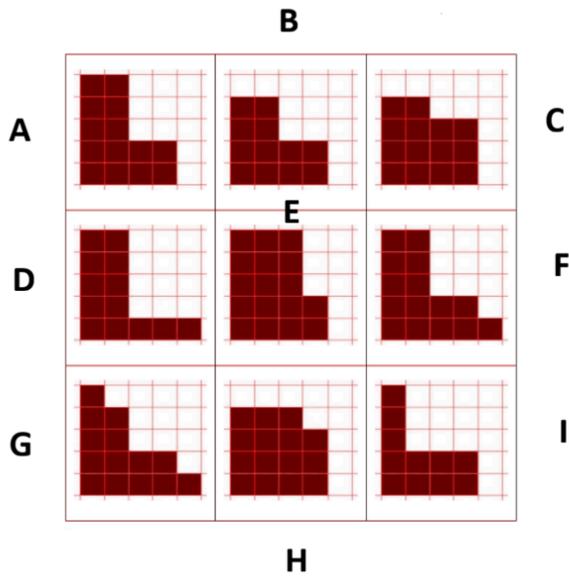
- कोणत्याही ओळीत डावीकडून उजवीकडे जाताना, आकारांचे क्षेत्रफळ वाढत गेले पाहिजे.
- कोणत्याही स्तंभात वरून खाली जाताना, आकारांची परिमिती वाढत गेली पाहिजे.
- मध्य स्तंभातील सर्व आकारांचे क्षेत्रफळ सारखेच असले पाहिजे.
- मध्य ओळीतील सर्व आकारांची परिमिती सारखीच असली पाहिजे.

या आकारांची मांडणी दिलेल्या 3 x 3 ग्रिड मध्ये करण्यासाठी तुम्ही लागू केलेले पद्धतशीर तर्क / गणितीय धोरण टप्प्याटप्प्यात स्पष्ट लिहा. तसेच, 3 x 3 ग्रिड मधील त्यांचे स्थान निश्चित करा.



**Solution:**

1. Label each shape.



2. Write down the areas and perimeters of each shape.

Shape	Area	Perimeter
A	14	18
B	12	16

C	14	16
D	13	20
E	17	18
F	15	20
G	14	20
H	15	16
I	11	18

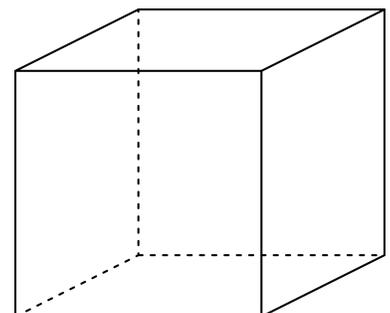
3. Three shapes that have equal area (and increasing perimeters) are C, A & G.  
Therefore, they must occupy the middle column.
4. Shapes B, I & D have areas less than 14. So, they must occupy the left-most column.
5. Shapes H, E & F have areas greater than 14. So, they must occupy the right-most column.

6. Shapes with a perimeter of 20 (D, G & F) must go in the bottom row, and those with a perimeter of 18 (I, A & E) must go in the middle row.
  7. Finally, those with a perimeter of 16 (B, C & H) must go in the top row.
  8. There is only one way this can be achieved.
- So, the solution is obtained by a process of elimination.

**Final Arrangement:**

				<b>AREA</b>		
				-	=	+
<b>PERIMETER</b>	-	<b>B</b> <small>Area = 12, Perimeter = 16</small>	<b>C</b> <small>Area = 14, Perimeter = 16</small>	<b>H</b> <small>Area = 15, Perimeter = 16</small>		
	=	<b>I</b> <small>Area = 11, Perimeter = 18</small>	<b>A</b> <small>Area = 14, Perimeter = 18</small>	<b>E</b> <small>Area = 17, Perimeter = 18</small>		
	+	<b>D</b> <small>Area = 13, Perimeter = 20</small>	<b>G</b> <small>Area = 14, Perimeter = 20</small>	<b>F</b> <small>Area = 15, Perimeter = 20</small>		

12. A positive integer is assigned to each of the six faces of a cube. At each of the eight vertices, we calculate and write the product of the integers assigned to the three faces that meet at that point. If the sum of these eight vertex products is 154, what is the sum of the six integers assigned to the faces?



एका घनाच्या सहा पृष्ठांवर एकेक धन पूर्णांक लिहिला आहे.

त्या घनाच्या आठ शिरोबिंदूंपैकी प्रत्येकावर त्या बिंदूतून जाणाऱ्या तीन पृष्ठांवरील अंकांचा गुणाकार करून मिळालेली संख्या लिहिली आहे. जर या आठ शिरोबिंदूंवरील सर्व गुणाकारांची बेरीज 154 असेल, तर त्या सहा पृष्ठांवरील पूर्णांकांची बेरीज किती असेल?

**Answer:** 20

**Solution:**

Let the six faces of the cube be represented by three pairs of opposite faces:

a and c (Opposite each other)

b and d (Opposite each other)

e and f (Opposite each other)

Each of the 8 vertices is formed by the intersection of three faces.

The sum of these products can be written as:

$$S = abe + abf + bce + bcf + cde + cdf + ade + adf = 154$$

$$\text{So, } ab(e + f) + bc(e + f) + cd(e + f) + ad(e + f) = 154$$

$$(e + f)(ab + bc + cd + ad) = 154$$

$$(e + f)(b(a + c) + d(c + a)) = 154$$

$$(e + f)(a + c)(b + d) = 154$$

To find the values of these sums, we find the prime factors of 154:  $154 = 2 \times 7 \times 11$

Since the faces are positive integers, each factor  $(a + c)$ ,  $(b + d)$ , and  $(e + f)$  must be at least 2.

Therefore,  $(e + f) = 2$ ,  $(a + c) = 7$  and  $(b + d) = 11$ .

Thus, the required sum of the integers assigned to the faces

$$= a + b + c + d + e + f = (e + f) + (a + c) + (b + d) = 20.$$

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