BMTSC, 2017 (Answer Key)

1. Q.1 Choose one correct alternative.

   1. (D) $-13, -3, 0, 2$
   2. (B) 20, $\dfrac{8}{10} = \dfrac{A}{25}$
      $\therefore A = 25 \times \dfrac{8}{10} = \dfrac{200}{10} = 20$
   3. (C) 30, $10 \cdot 5 = \dfrac{x}{100} \times 35$
      $\therefore x = \dfrac{10 \cdot 5 \times 100}{35} = \dfrac{1050}{35} = 30$
   4. (B) 22,
      $24 \times \left(2\frac{3}{4} - 1\frac{5}{16}\right) = 24 \times \left(\frac{11}{4} - \frac{11}{6}\right) = 24 \times \left(\frac{33-22}{12}\right) = 24 \times \frac{11}{12} = 22.$
   5. (C) 84, $22^2 - 20^2 = 484 - 400 = 84.$

      Let $2n$ and $2n+2$ be two consecutive even numbers. Then $(2n+2)^2 - (2n)^2 = 4n^2 + 8n + 4 - 4n^2 = 8n + 4$ i.e. difference between squares of two consecutive even numbers leaves remainder 4 when divided by 8. Only such number form the options is 84.

   6. (B) 8

      $8A2$ is divisible by 9.

      $\therefore 8 + A + 2 = 10 + A$ is multiple of 9.

      $\therefore A = 8.$

   7. (B) 50

      $x^\circ + 2x^\circ = 150^\circ$

      $\therefore 3x^\circ = 150^\circ$

      $\therefore x^\circ = \dfrac{150^\circ}{3} = 50^\circ$

8. (B) 19

$$1 \star 2 = 1 + 2^2 = 5$$
$$2 \star 3 = 2 + 3^2 = 11$$
$$\therefore 3 \star 4 = 3 + 4^2 = 19$$

9. (B) 56

2^{nd} chair is in front of 30^{th} chair

$\therefore$ 1^{st} chair is in frant of 29^{th} chair

$\therefore$ Between 1^{st} chair & 29^{th} chair there are 27^{th} chairs on both sides.

$\therefore$ Total number of chairs is $27 + 27 + 1^{\text{st}}$ chair $+29^{\text{th}}$ chair $= 56$.

10. (D) 51

Windows are at seat numbers

$1, 5, 6, 10, 11, 15, 16, 20, 21, 25, 26, 30, 31, 35, 36, 40, 41, 45, 46, 50, 51.$

11. (D) 3

$$303 = \underline{3} \times 101$$
$$33 = \underline{3} \times 11$$
$$330 = \underline{3} \times 2 \times 5 \times 11$$
$$3003 = \underline{3} \times 1001$$
$$GCD = 3$$

12. (B) 20°

Let $m(\angle ABE) = m(\angle EBC) = x^\circ$

$\therefore m(\angle ABC) = m(\angle ACB) = 2x^\circ$

Then, $m(\angle EBC) + m(\angle ECB) = 120^\circ$ gives

$x^\circ + 2x^\circ = 120^\circ$

$\therefore x^\circ = 40^\circ$

In $\triangle ABE, m(\angle ABE) + m(\angle AEB) + m(\angle BAE) = 180^\circ$ gives,

$\therefore x^\circ + 120^\circ + m(\angle BAC) = 180^\circ$

$\therefore m(\angle BAC) = 180^\circ - 120^\circ - x^\circ = 180^\circ - 120^\circ - 40^\circ - 20^\circ.$

13. (D) 2011

    May 1, 2005 is Friday

    365 days = 52 weeks, 1 day & 366 days = 52 weeks, 2 days

May 1, 2006	May 1, 2007	May 1, 2008	May 1, 2009	May 1, 2010	May 1, 2011
Saturday	Sunday	Tuesday (Leap Year)	Wednesday	Thrusday	Friday

14. (B) 4

15. (C) 25%

    Out of 6 faces of newly obtained solid, 4 faces are painted blue and each of then have same surface area which is half of the surface area of a face of original cube.

    Each of the remaining 2 faces of newly obtained solid have surface area equal to that of a face of original cube. Out of these 2 faces, one is painted blue and other is colourless.

    $\therefore$ total surface area of newly obtained solid

    $$= 4 \times \left(\begin{array}{c} \frac{1}{2} \times \text{surface area of a face} \\ \text{of original cube} \end{array} \right) + 2 \times \left(\begin{array}{c} \text{surface area of a face} \\ \text{of original cube} \end{array} \right)$$

    = 4 times surface area of a face of original cube.

    Area of colourless surface of newly obtained solid.

    = surface area of a face of original cube.

    $\therefore$ % of surface area of each of new solids not painted blue is $\frac{1}{4} \times 100 = 25\%$.

Q.2. Solve

1. False.

   12 is divisible by 4 and 6 but is not divisible by 24.

2. $90 = 2 \times 3 \times 3 \times 5$

   $15 = 3 \times 5$

   Other number must be divisible by 2 and 9

$\therefore$ Smallest possible value for the other number is 18.

3. Draw segment MN parallel to BC passing through E.

   $\therefore m(\angle AME) = m(\angle DNE) = 90^\circ$

   Figure.

   In $\triangle AME, m(\angle AME) + m(\angle MAE) + m(\angle AEM) = 180^\circ$

   $\therefore 90^\circ + 30^\circ + m(\angle AEM) = 180^\circ$

   $\therefore m(\angle AEM) = 180^\circ - 90^\circ - 30^\circ = 60^\circ \quad (1)$

   In $\triangle DNE, m(\angle DNE) + m(\angle NDE) + m(\angle DEN) = 180^\circ$

   $\therefore 90^\circ + 80^\circ + m(\angle DEN) = 180^\circ$

   $\therefore m(\angle DEN) = 180^\circ - 90^\circ - 80^\circ = 10^\circ \quad (2)$

   Now, $m(\angle AEM) + m(\angle AED) + m(\angle DEN) = 180^\circ$

   $\therefore 60^\circ + m(\angle AED) + 10^\circ = 180^\circ$

   $\therefore m(\angle AED) = 180^\circ - 60^\circ - 10^\circ = 110^\circ$

   (from equtions (1) and (2))

4. If 8 pages are read every day, total no. of pages read in whole of

   June $= 30 \times 8 = 240$.

   In book has to be completed in 12 days, the no. of pages should be read every day $= \dfrac{240}{12} = 20$.

5. Let $l =$ length, $b =$ breadth

   As, perimeter and area should be equal,

   $2(l + b) = lb$

   $\therefore 2l + 2b = lb$

   $\therefore 2l - lb + 2b = 0$

   $\therefore l(2 - b) + 2b = 0$

   $\therefore l = -\dfrac{2b}{2-b} = \dfrac{2b}{b-2}$.

   As b is breadth, it cannot be zero or negative.

   If $b = 1,\ l = \frac{2}{-1} = -2$ which is not possible as l is length.

   If $b = 2,\ l$ is not defined.

If $b = 3,\ l = \dfrac{2 \times 3}{3-2} = \dfrac{6}{1} = 6$

If $b = 4,\ l = \dfrac{2 \times 4}{4-2} = \dfrac{8}{2} = 4$

If $f \geq 5,\ l = \dfrac{2b}{b-2}$ is not as integer.

$\therefore l = 6, b = 3$ and $l = 4, b = 4$ are two possible dimensions.

6. Figuer

   Let $ABCDEF$ be hexagon whose 5 interior angles have measures

   $130^\circ, 120^\circ, 105^\circ, 140^\circ$ and 100°.

   Let 0 be any point inside hexagon $ABCDEF$.

   Join OA, OB, OC, OD, OE and OF.

   Sum of interior angles of a triangle is 180°.

   There are 6 triangles $\triangle AOB, \triangle BOC, \triangle COD, \triangle DOE,\ \triangle EOF$ and $\triangle FOA$

   in hexagon $ABCDEF$.

   Sum of all angles of all the 6 triangles is $180^\circ \times 6 = 1080^\circ$.

   Also, $m(\angle AOB) + m(\angle BOC) + m(\angle COD) + m(\angle DOE)$

   $+ m(\angle EOF) + m(\angle FOA) = 360^\circ$

   Thus, sum of all interior angles of hexagon is $1080^\circ - 360^\circ = 720^\circ$.

   Sum of 5 given interior angles is $130^\circ + 120^\circ + 105^\circ + 140^\circ + 100^\circ = 595^\circ$.

   $\therefore$ measure of remaining interior angle is $720^\circ - 595^\circ = 125^\circ$.

7. Square $ABCD$ has side length 'a'

   $\therefore$ Area of square $ABCD = a^2$.

   M, N, P and Q are midpoints of AB, AD, BC and CD resp.

   $\therefore l(AN) = l(AM) = l(PC) = l(CQ) = \dfrac{a}{2}$.

   $\therefore A(\triangle AMN) = A(\triangle PCQ) = \dfrac{1}{2} \times \dfrac{a}{2} \times \dfrac{a}{2} = \dfrac{a^2}{8}$.

   $\therefore A(\triangle AMN) + A(\triangle PCQ) = \dfrac{a^2}{8} + \dfrac{a^2}{8} = \dfrac{a^2}{4}$.

   Thus, area of shaded region

   $= A(sq.ABCD) - [A(\triangle AMN) + A(\triangle PCQ)]$

   $= a^2 - \dfrac{a^2}{4} = \dfrac{3a^2}{4}$.

8. Case 1: Assume that Rajesh had made a correct statement.

   $\therefore$ Suresh had made incorrect statement.

   Thus, Chintu won the contest.

   But then Mahesh's statement will be correct which is not possible since only one of them had made correct statement.

   $\therefore$ Rajesh's statement is incorrect.

   Case 2: Assume that Suresh had made a correct statement.

   $\therefore$ Chintu did not win.

   Now, Rajesh's statement has to be incorrect.

   $\therefore$ neither Chintu nor Raju won the contest. (1)

   Also, Mahesh's statement has to be incorrect.

   $\therefore$ either Mini or Raju won the contest. (2)

   Thus, combining statements (1) and (2), we can conclude that Mini won the contest and Suresh's statement is correct as there is no logical conflict in this case.

   Case 3: Assume that Mahesh has made a correct statement.

   $\therefore$ Suresh had made incorrect statement.

   Thus, Chintu won the contest.

   But then Mahesh's statement will be correct which is not possible since only one of them had made correct statement.

   $\therefore$ Mahesh's statement is incorrect.

   Thus, from all the three cases, Mini won the contest and Suresh had made correct statement.

Q.3 Write the answer with justification.

1. Let GCD of 'n' and 36 be x.

   $\therefore$ LCM of 'n' and 36 is $4x$.

   Product of two nos. $=$ GCD of two nos. $\times$ LCM of two nos.

$\therefore n \times 36 = x \times 4x = 4x^2$

$\therefore 9n = x^2$

$\therefore 9n$ is a perfect square.

$\therefore n$ has to be a perfect square.

If $n = 1$, GCD $= 1$ and LCM $= 36$

If $n = 4$, GCD $= 4$ and LCM $= 36$

For above two values of n, LCM is not 4 times GCD.

If $n = 9$, GCD $= 9$ and LCM $= 36$

Thus, LCM is 4 times GCD.

If $n = 16$, GCD $= 4$ and LCM $= 144$

If $n = 25$, GCD $= 1$ and LCM $= 25 \times 36$

If $n = 36$, GCD $= 36$ and LCM $= 36$

If $n = 49$, GCD $= 1$ and LCM $= 36 \times 49$

If $n = 64$, GCD $= 4$ and LCM $= 576$

If $n = 81$, GCD $= 9$ and LCM $= 324$

If $n = 100$, GCD $= 4$ and LCM $= 900$

If $n = 121$, GCD $= 1$ and LCM $= 121 \times 36$

For all of the above values of n, LCM is not 4 times GCD.

If $n = 144$, GCD $= 36$ and LCM $= 144$

Thus, LCM is 4 times GCD.

Maximum possible value of GCD of 'n' and 36 is 36.

$\therefore$ Maximum possible value of $n = \frac{x^2}{9} = \frac{36^2}{9} = 144$.

Thus, only possible values of n are 9 and 144.

2. Let 'd' be the distance that catherine travels.

   Abdul travels thrice the distance that catherine travels.

   $\therefore$ distance travelled by Abdul is '$3d$'

   Abdul travels twice the distance that Binoy travels.

   $\therefore$ distance travelled by Binoy is '$\frac{3d}{2}$'.

Let 's' be the speed of catherine.

Catherine's speed is $\frac{1}{3}$ of Abdul's speed.

$\therefore$ speed of Abdul is '$3s$'

Catherine's speed is $\frac{1}{2}$ of Binoy's speed.

$\therefore$ speed of Binoy is '$2s$'

Thus, Abdul requires $\dfrac{3d}{3s}$ time to reach his destination,

Catherine requires $\dfrac{d}{s}$ time to reach her destination and Binoy requires $\dfrac{\frac{3d}{2}}{2s} = \dfrac{3d}{4s}$ time to reach his destination.

Thus, Binoy takes least time to reach his destination and hence he reaches his destination first.

3. Given no. has 11 digits.

   By deleting 10 digits, we will get a no. with 9 digits. Now, the resultant 9 digit no. is as small as possible, if its left most digit (most significant digit) is as small as possible.

   In the given no., there are exactly 10 digits to the left of digit '0'.

   $\therefore$ if we remove all the 10 digits to the left of '0' from the given no., we get '011121314' as the smallest possible no.

4. Let the jars numbered 1, 2 and 3 have capacities 3 litres, 5 litres and 8 litres respectively.

   Initially the configuration is $(0, 0, 8)$.

Solution 1:				Solution 2:		
	Jar 1 (3 lit.)	Jar 2 (5 lit.)	Jar 3 (8 lit.)	Jar 1 (3 lit.)	Jar 2 (5 lit.)	Jar 3 (8 lit.)
Initial Condition	0	0	8	0	0	8
	3	0	5	0	5	3
	0	3	5	3	2	3
	3	3	2	0	2	6
	1	5	2	2	0	6
	1	0	7	2	5	1
	0	1	7	3	4	1
	3	1	4	0	4	4
	0	4	4			

5. Suppose Raghu has x no. of flowers Initially and he offers y no. of them at temple 1.

   $\therefore$ He remains with $x - y$ no. of flowers.

   When he goes to temple 2, remaining $x - y$ no. of flowers get doubled.

   So, he has $2x - 2y$ no. of flowers. He offers y no. of flowers at temple 2.

   $\therefore$ He remains with $2x - 2y - y$ no. of flowers. Again, the remaining $2x - 3y$ no. of flowers get doubled when he goes to temple 3.

   So, he has $4x - 6y$ no. of flowers. He offers y no. of flowers at temple 3.

   Now, no flowers are left with him.

   $\therefore 4x - 6y - y = 0$

   $\therefore 4x = 7y$

   $\therefore \dfrac{x}{y} = \dfrac{7}{4}$

   i.e. ratio of initial no. of flowers to no. of flowers offered at each temple is 7 : 4.

If initially he has 7 no. of flowers, then he offers 4 flowers in each temple.

If initially he has 14 no. of flowers, then he offers 8 flowers in each temple.

Q.4. Write the answers with justification.

1. (a) Perimeter $(\square APTS)+$ Perimeter $(\square PBQT)+$ Perimeter $(\square QCRV)+$ Perimeter $(\square VRDS) = 2$ Perimeter $(\square ABCD)$

    $\therefore 60 + 140 + 130 + 110$ units $= 2$ Perimeter $(BoxABCD)$

    $\therefore$ Perimeter $(\square ABCD) = \dfrac{440}{2}$ units $= 220$ units.

   (b) Perimeter $(\square APTS) = 60$ units.

    i.e. $2l(AP) + 2l(AS) = 60$ units.

    Given that $l(AS) = 20$ units.

    $\therefore 2l(AP) = 60 - 2l(AS) = 60 - 40 = 20$ units.

    $\therefore l(AP) = 10$ units. (1)

    Perimeter $(\square PBQT) = 140$ units.

    i.e. $2l(PT) + 2l(PB) = 140$ units.

    Given that $l(AS) = l(PT) = l(BQ) = 20$ units.

    $\therefore 2l(PB) = 140 - 2l(PT) = 140 - 40 = 100$ units.

    $\therefore l(PB) = 50$ units. (2)

    From equations (1) and (2),

    $l(AB) = l(AP) + l(PB) = 10 + 50 = 60$ units.

    Now, from part a),

    Perimeter $(\square ABCD) = 220$ units.

    i.e. $2l(AB) + 2l(BC) = 220$ units.

    $\therefore 2l(BC) = 220 - 2l(AB) = 220 - 120 = 100$ units.

    $\therefore l(BC) = 50$ units.

    Thus, length of rectangle $ABCD$ is 60 units and its breadth is 50 units.

2. (a)

Box 1	Box 2	Box 3
4	3	0

If we remove 1 coin from box 1, then configuration will be $(3, 6, 0)$

If we remove 1 coin from $60x, 2$ then we can add 3 coins to any one of box 1 and box 3.

$\therefore$ there are two possible configuration $(7, 2, 0)$ and $(4, 2, 3)$.

(b) In a move, we remove 1 coin from a box and add 3 coins to any of its adjacent boxes.

Thus, in each move we add $-1 + 3 = 2$ coins in the sum of coins in all the three boxes.

If we start with configuration $(1, 1, 1)$ sum of coins in all the three boxes is 3, which is odd.

In every move, 2 no. of coins will be added. So, we will always get sum of coins in all the three boxes to be odd.

Hence, we can not get 50 as sum of coins in all the three boxes.

(c) If we begin with configuration $(1, 1, 1)$ and suppose that it is possible to get configuration $(10, 2, 3)$, then starting with sum of coins in all the three boxes $1+1+1 = 3$, we end up with sum of coins in all the three boxes as $10+2+3 = 15$.

$\therefore$ we wish to add $(15 - 3) = 12$ coins which will need 6 moves. (1)

Now, to get 10 coins in 1st box initially containing 1 coin, we will require atleast 3 moves in which we remove a coin from box 2 and add 3 coins to box 1. (2)

This will require at least 3 coins in box 2. At the end, we need 2 coins in box 2 (which initially contains 1 coin).

$\therefore$ During the process, we require at least $3 + 2 - 1 = 4$ coins in box 2. This needs at least 2 moves to insert $3 + 3 = 6$ coins in box 2 and at least 2 moves to remove 2 coins from box 2, so as to get $6 - 2 = 4$ coins in box 2. (3)

But, all these moves add up to $3 + 2 + 2 = 7$, which is not possible.

(from (1), (2), (3))

$\therefore$ We can not get configuration $(10, 2, 3)$ if we start with $(1, 1, 1)$.

3. (a) If there are 'n' stations on a rail tract and if we add 1 more station on the track, then we need to print additional tickets from each previously existing station to newly added station and from newly added station to each previously existing station. Thus, we need to print $2 \times n$ no. of additional types of tickets.

   $\therefore$ If 1 station is added on a rail track which already has 10 stations, $2 \times 10 = 20$ additional types of rail tickets should be printed.

   (b) There are 13 stations on a rail track.

   If 1 station is added $2 \times 13 = 26$ new types of rail tickes should be printed.

   Now, there are 14 stations on the track. If 1 more station is added, $2 \times 14 = 28$ new types of rail tickets should be printed.

   $\therefore$ If we add two stations on the rail track which already has 13 stations, we need to print $26 + 28 = 54$ types of rail tickets additionally.

   (c) If there are 'n' stations on a rail track and if we add 1 more station on the track, then we need to print '$2n$' types of rail tickets additionally.

   Given that 44 new types of rail tickets should be printed.

   $\therefore 44 = 2n$ gives $n = 22$.

   Thus, 22 stations are already present on the rail track and 1 station is newly added.

   One more solution is:

   4 stations are already present on the rail track and 4 stations are newly added.