**BHASKARACHARYA PRATISHTHANA, PUNE**
**Bhaskaracharya Mathematics Talent Search Competition, 2019**
**Solutions**

Q.1.

1. Option (B) $\frac{1}{3} - \frac{1}{4}$.
2. Option (C) 79.
3. Option (A) 3.
4. Option (C) 4.
   $\frac{1/2}{100} \times 800 = \frac{1}{2} \times \frac{1}{100} \times 800 = 4.$
5. Option (B) $11\frac{1}{9}\%$.
   $\frac{100}{900} \times 100 = \frac{100}{9} = 11\frac{1}{9}\%.$
6. Option (D) $2\frac{1}{3} + 5\frac{1}{2} + 1\frac{1}{6}$.
7. Option (B) $\frac{1}{6}$.
   $p : q = 3 : 8$ and $q : r = 4 : 9 = 8 : 18$. Thus, $p : r = 3 : 18 = 1 : 6$.
8. Option (C) 19.
   $\frac{x-5}{7} = 2$
   $\therefore x - 5 = 14$
   $\therefore x = 14 + 5 = 19.$
9. Option (B) 4.
10. Option (B) 10.
    Total number of squares of size 1cm × 1cm = 9 × 4 = 36.
    Out of these, 6 squares are removed. Thus, 1cm × 1cm squares to be covered are 30.
    Given shape consists of 3 squares of size 1cm × 1cm.
    $\therefore$ Number of shapes required are $\frac{30}{3} = 10$.
11. Option (A) 42.
    Let $x$ be length of the pole.
    $\therefore \frac{x}{2} + \frac{x}{3} + 7 = x$
    $\therefore 3x + 2x + 42 = 6x.$

    $\therefore x = 42.$
12. Option (C) 10.
    340, 430, 390, 930, 490, 940, 304, 904, 394, 934.

13. Option (B) 4.
14. Option (D) 900.
    The number of students should be multiple of 5, 10, 15 and it should be a perfect square also. Only such number from options is 900.
15. Option (A) increase by 2 cm.
    Everyone has become taller, so there will be increase in the average height.

$$\begin{aligned}\text{Increase in Average Height} &= \frac{\text{Total Increase in Height}}{\text{Number of Friends}} \\ &= \frac{1+3+2+2}{4} \\ &= \frac{8}{4} = 2.\end{aligned}$$

Q.2.

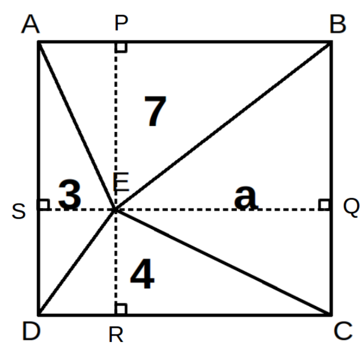
1. A natural number that is divisible by all the three numbers 5, 6 and 9 must be a multiple of their LCM.
   LCM of 5, 6 and 9 is 90.
   3-digit multiples of 90 are $180, 270, 360, 450, 540, 630, 720, 810, 900, 990$.
   Thus, there are 10 such numbers.
2. The ratio of the number of sweet lemons to the number of oranges is 1 : 2 which is same as 2 : 4.
   The ratio of the number of sweet lemons to the number of apples is given as 2 : 3.
   Thus, the combined ratio of the number of sweet lemons to the number of oranges to the number of apples is 2 : 4 : 3.
   The number of apples are given to be 9.
   Hence, the common multiple in the combined ratio must be 3.
   Therefore, the number of oranges in the bag are $4 \times 3 = 12$.
3. Usha takes 24 minutes to go to school and come back home from school when she runs both ways.
   Thus, one way it takes her 12 minutes when she runs.
   She takes 40 minutes together to go to school by walk and run back home from school.
   Therefore, when she walks one way, it takes her $40 - 12 = 28$ minutes.
   Hence, it would take her $28 \times 2 = 56$ minutes to walk both ways.
4. In right angled triangle $ABC$,
   $m\angle ABC + m\angle BAC + m\angle BCA = 180°$
   $\therefore 90° + m\angle BAC + m\angle BCA = 180°$
   $\therefore m\angle BAC + m\angle BCA = 90°$ (1)
   $OA$ and $OC$ are angle bisectors of $\angle A$ and $\angle C$ respectively.
   $\therefore m\angle OAC = \frac{1}{2} m\angle BAC$ and $m\angle OCA = \frac{1}{2} m\angle BCA$.

In $\triangle AOC, m\angle OAC + m\angle OCA + m\angle AOC = 180°$
$\therefore \frac{1}{2}m\angle BAC + \frac{1}{2}m\angle BCA + m\angle AOC = 180°$
$\therefore \frac{1}{2}(m\angle BAC + m\angle BCA) + m\angle AOC = 180°$
$\therefore \frac{1}{2} \times 90° + m\angle AOC = 180°$ (from equation (1))
$\therefore m\angle AOC = 180° - 45° = 135°$.

5. LCM of $a, c = 12$.
Thus, possibilities for $a$ and $c$ are $1, 2, 3, 4, 6, 12$.
Following values of $a, b, c$ satisfy given conditions.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $a$ | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| $b$ | 6 | 6 | 6 | 4 | 4 | 3 | 2 |
| $c$ | 3 | 2 | 1 | 2 | 1 | 1 | 1 |

6. Let $P, Q, R, S$ be the foot of the perpendiculars drawn from point $E$ to the sides $AB, BC, CD, DA$ respectively.




Now, $A(\triangle AEB) + A(\triangle CED) = \frac{1}{2} \times EP \times AB + \frac{1}{2} \times ER \times CD$
and $A(\triangle AED) + A(\triangle BEC) = \frac{1}{2} \times ES \times DA + \frac{1}{2} \times EQ \times BC$
As, $\square ABCD$ is a square, $AB = BC = CD = DA$.
$\therefore A(\triangle AEB) + A(\triangle CED) = \frac{1}{2} \times (EP + ER) \times AB$
and $A(\triangle AED) + A(\triangle BEC) = \frac{1}{2} \times (ES + EQ) \times AB$
Also, $EP + ER = ES + EQ = AB$
$\therefore A(\triangle AEB) + A(\triangle CED) = A(\triangle AED) + A(\triangle BEC)$
$\therefore 7 + 4 = 3 + a$
$\therefore a = 8$

7. Suppose the statement made by thief is True.
As, the statement made by him is 'True', the day must be one of Monday, Wednesday or Friday.
Also, as the statement is True, he will tell the truth the next day. That means, the thief tells the truth on two consecutive days of the week which is not possible.
Thus, the statement made by the thief must be False.

As, the statement made by him is 'False', the day must be one of Tuesday, Thursday, Saturday or Sunday.
Also, as the statement is False, he will tell lie the next day. That means, the thief lies on two consecutive days of the week, which is possible if the statement is made on Saturday.

8. Figure 1 can be formed as shown:

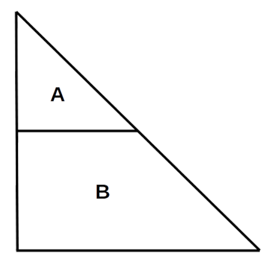
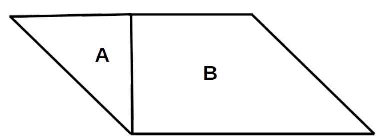



Figure 2 cannot be formed because area of figure 2 is 4 sq. units and area of shapes $A$ and $B$ together is 2 sq. units.
Figure 3 can be formed as shown:




Q.3. 1. Let $abc$ be 3-digit number having distinct digits.
As, the product of the digits is 24, possibilities for $a, b, c$ are $1, 2, 12$ or $1, 3, 8$ or $1, 4, 6$ or $2, 3, 4$.
Also, the sum of the digits is an even number.
Thus, possibilities for $a, b, c$ are $1, 3, 8$.
As, the number should be smallest, $a = 1, b = 3, c = 8$.
i.e. 138 is the smallest 3-digit number satisfying given conditions.

2. Suppose a girl requires $G$ number of days and a boy requires $B$ number of days to paint a room.
$\therefore$ part of room painted by a girl in one day $= \dfrac{1}{G}$ and part of room painted by a boy in one day $= \dfrac{1}{B}$.
A girl and a boy require 6 days to paint a room.
i.e. a girl and a boy can paint $\dfrac{1}{6}$ part of a room in one day.

i.e. $\dfrac{1}{G}+\dfrac{1}{B}=\dfrac{1}{6}$.

$$\therefore \frac{1}{G}=\frac{1}{6}-\frac{1}{B}. \qquad (1)$$

3 girls and 4 boys require 5 days to paint 3 rooms.

$\therefore \dfrac{3}{G}+\dfrac{4}{B}=\dfrac{1}{5}\times 3.$

$$\therefore 3\times\left(\frac{1}{6}-\frac{1}{B}\right)+\frac{4}{B}=\frac{3}{5}. \qquad \text{(by equation (1))}$$

$\therefore \dfrac{1}{2}-\dfrac{3}{B}+\dfrac{4}{B}=\dfrac{3}{5}.$

$\therefore \dfrac{1}{B}=\dfrac{3}{5}-\dfrac{1}{2}=\dfrac{6-5}{10}=\dfrac{1}{10}.$

$\therefore B=10.$

$\therefore \dfrac{1}{G}=\dfrac{1}{6}-\dfrac{1}{B}=\dfrac{1}{6}-\dfrac{1}{10}=\dfrac{5-3}{30}=\dfrac{2}{30}=\dfrac{1}{15}.$

$\therefore G=15.$

Suppose 6 girls and 5 boys require '$n$' number of days of paint 9 rooms.

$\therefore \dfrac{6}{G}+\dfrac{5}{B}=\dfrac{1}{n}\times 9.$

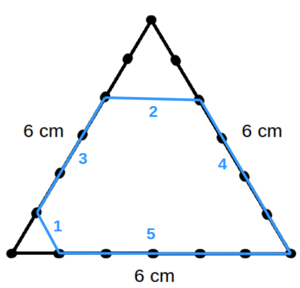
$\therefore \dfrac{6}{15}+\dfrac{5}{10}=\dfrac{9}{n}.$

$\therefore \dfrac{9}{n}=\dfrac{12+15}{30}=\dfrac{27}{30}.$

$\therefore n=\dfrac{30\times 9}{27}=10.$

Thus 6 girls and 5 boys will require 10 days to paint 9 rooms.

3. Observe the figure:



4. Let the amount of oil in glass $A$ be $x$ ml.
$\therefore$ Amount of water in glass $A = 100 - x$ ml.
Amount of oil in glass $B = 2x$ ml.
Amount of water in glass $B = 100 - 2x$ ml.
Amount of water in glass $C = \dfrac{100-2x}{2}$ ml $= 50 - x$ ml.
Amount of oil in glass $C = 100 - (50 - x)$ ml $= 50 + x$ ml.
When all the glasses are poured into a single large container, the resulting mixture has equal amount of water and oil.

$$\therefore x + 2x + 50 + x = 100 - x + 100 - 2x + 50 - x$$
$$\therefore 4x + 50 = 250 - 4x$$
$$\therefore 8x = 200$$
$$\therefore x = 25$$

Thus, amout of oil in glasses $A, B, C$ are 25 ml, 50 ml, 75 ml respectively and amount of water in glasses $A, B, C$ are 75 ml, 50 ml, 25 ml, respectively.

5. Surface area of the box $= 2(lb + bh + lh)$.
Breadth and height of the box are equal.
$\therefore$ Surface area $= 2(lb + b^2 + lb) = 2(2lb + b^2)$

$$\therefore 2(2lb + b^2) = 72$$
$$\therefore 2b(2l + b) = 72$$
$$\therefore b(2l + b) = 36$$

Breadth should be minimum so that length will be maximum.
If $b = 1, 2l + b = 36$
$\therefore 2l = 36 - b = 36 - 1 = 35$
$\therefore l = \dfrac{35}{2}$, not a natural number
If $b = 2, 2l + b = \dfrac{36}{2} = 18$
$\therefore 2l = 18 - b = 18 - 2 = 16$
$\therefore l = \dfrac{16}{2} = 8$
Thus, the largest possible length of the box is 8 units.

Q.4 1. (a) Area of given figure

$$= A(\triangle ABC) + A(\triangle FIH) + A(\triangle CDE) + A(\square AHIC)$$
$$= \frac{1}{2} \times 4 \times 3 + \frac{1}{2} \times 12 \times 5 + \frac{1}{2} \times 8 \times 6 + 4 \times 5$$
$$= 6 + 30 + 24 + 20$$
$$= 80 \text{ sq. cm.}$$

(b)

$$
\begin{aligned}
A(\square EGHI) &= A(\triangle FIH) - A(\triangle FEG)\\
&= \frac{1}{2} \times 12 \times 5 - \frac{1}{2} \times 6 \times 2.5\\
&= 30 - 7 \cdot 5\\
&= 22 \cdot 5 \text{ sq. cm}
\end{aligned}
$$

(c) $\triangle FIH$ has area as 30 sq. cm and perimeter as 30 cm.

OR

$\triangle CDE$ has area as 24 sq. cm and perimeter as 24 cm.

(d) Region $GEICAH$ has area as $22{\cdot}5+20 = 42{\cdot}5$ sq. cm. and perimeter as $2 \cdot 5 + 6 + 4 + 5 + 4 + 6 \cdot 5 = 28$ cm.

2. The given sequence of numbers is $9, 12, 15, 18, 21, \cdots$. It can also be written as $3(3), 3(4), 3(5), 3(6), 3(7), \cdots$ .

(a) All numbers in the sequence are multiples of 3.
Thus, perfect squares in the sequence must be multiples of 9.
Thus, first three perfect squares are $9 \times 1^2,\ 9 \times 2^2,\ 9 \times 3^2$.

(b) $100^{\text{th}}$ perfect square is $9 \times (100)^2 = 90000$

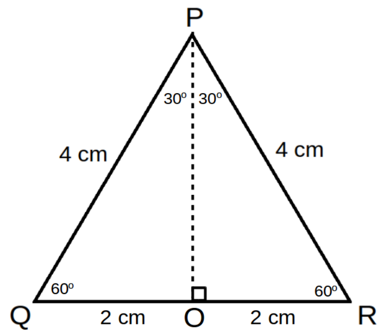
(c) Sum of first 100 terms in the sequence

$$
\begin{aligned}
&= 3 \times (3) + 3 \times (4) + 3 \times (5) + \cdots + 3 \times (102)\\
&= 3 \times (3 + 4 + 5 + \cdots + 102)\\
&= 3 \times [(1 + 2 + 3 + \cdots + 102) - (1 + 2)]\\
&= 3 \times \left[\frac{102 \times 103}{2} - 3\right]\\
&= 3 \times (5253 - 3)\\
&= 3 \times 5250\\
&= 15750.
\end{aligned}
$$

3. (a) By Pythagoras' Theorem,

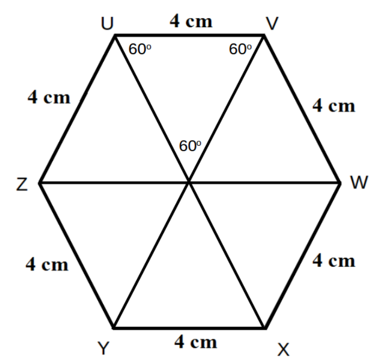
$$
\begin{aligned}
LM^2 + MN^2 &= LN^2\\
\therefore LM^2 &= 4^2 - 2^2 = 12\\
\therefore LM &= \sqrt{12}.
\end{aligned}
$$

$$A(\triangle LMN) = \tfrac{1}{2} \times MN \times LM = \tfrac{1}{2} \times 2 \times \sqrt{12} = \sqrt{12} \text{ sq.cm.}$$




(b) Draw perpendicular from point $P$ to side $QR$ which meets $QR$ at point $O$.
As $\triangle PQR$ is equilateral triangle, $PO$ bisects $QR$.
Also, $PO$ bisects the angle $QPR$.
$\therefore m\angle QPO = m\angle RPO = 30°$.
Thus, $A(\triangle PQR) = 2A(\triangle POR) = 2A(\triangle LMN) = 2\sqrt{12}$ sq.cm.

(c) A regular hexagon can be divided into 6 equilateral triangles of same size.




$$\begin{aligned}\therefore A(UVWXYZ) &= 6 \times A(\triangle PQR)\\ &= 6 \times 2\sqrt{12}\\ &= 12\sqrt{12} \text{ sq. cm.}\end{aligned}$$