

Regional Mathematical Olympiad- 2006

Maharashtra and Goa Region

3rd December 2006

Max. Marks: 100

Time : 4 hours

- N.B.(i) There are 8 questions. All questions are compulsory.
(ii) Mathematical reasoning will be taken into consideration while assessing the answer.
(iii) Figures to the right indicate full marks for the question.

1. Let x, y, z be positive real numbers such that $xyz = 1$. If $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq x + y + z$, prove that

$$\frac{1}{x^k} + \frac{1}{y^k} + \frac{1}{z^k} \geq x^k + y^k + z^k,$$

for every positive integer k . [10]

2. Find all integers m such that $m + 3$ and $m^2 + 3m + 3$ are perfect cubes. [10]
3. Each year 8 subjects are taught by 4 teachers in a school. Every teacher teaches two subjects. At the end of this year they will meet to decide the course allotment for the next year. Find the number of ways in which the course distribution can be done so that each teacher teaches two courses and each teacher teaches at least one subject different from the subjects which he taught this year. [10]
4. Let C be a point on the circle with centre O and radius r . Chord AB of length r is parallel to radius OC . Let the line AO cut the circle in E and the tangent to the circle at C in F . If the chord BE cuts OC in L and if AL cuts CF in M , find the ratio $\frac{CF}{CM}$. [10]

5. In the set of complex numbers solve the system of equations

$$\begin{aligned}x(x - y)(x - z) &= 3, \\y(y - x)(y - z) &= 3, \\z(z - x)(z - y) &= 3.\end{aligned}\tag{15}$$

6. An 8×8 board is divided into unit squares. Each unit square is painted red or blue. Find the number of ways of doing this so that each row and each column has odd number of blue squares.
[15]

7. Find all natural numbers x, y, z such that

$$\sqrt{\frac{2006}{x+y}} + \sqrt{\frac{2006}{y+z}} + \sqrt{\frac{2006}{z+x}}$$

is a natural number.
[15]

8. Consider circle with centre O and radius OA . Let C be a point on radius OA . Let P be a variable point on the circle. Join P and C . Q is a point on the circle such that P and Q are on the same side of line OA and $\angle PCO = \angle QCA$. Find the locus of the point of intersection of the line PQ and the line OA .
[15]

