Advanced Training in Mathematics Schools

Funded by
National Board for Higher Mathematics

Annual Foundation Schools (AFS)

Objectives of AFS

Basic knowledge in algebra, analysis and topology forms the core of all advanced instructional schools organized in this programme. The objective of the Annual Foundation Schools, to be offered in Winter and Summer every year, is twofold:

- To bring up students with diverse backgrounds to a common level.
- To identify those who are fit for further training.

Any student who wishes to attend the advanced instructional schools is strongly encouarged to enroll in the Annual Foundation Schools first.

Format of AFS

The topics listed in the syllabi will be quickly covered in the lectures. There will be intensive problem sessions in the afternoons. The objective is not just to cover the syllabus prescribed, but to inculcate the habit of problem solving. However, the participants will be asked to study all the topics in the syllabus at home since the syllabi of these schools will be assumed in all the advanced instructional schools devoted to individual subjects.

Participants in AFS

These schools will admit **40 students** in their first and second years of Ph. D. programme, a few students of M. Sc. (II Year), 5 university lecturers and college teachers who lack the knowledge of basic topics covered in these schools. A participant who has attended AFS-I and II will never be allowed to attend these again.
The Programme Director will invite an eminent mathematician to deliver a series of lectures for one week called *Unity of Mathematics Lectures.* These lectures will be at the level of the courses being taught and they will be devoted to topics which involve several diverse areas of mathematics.

Daily Programme

Daily Programme for Annual Foundation School-I					
Time	9.00-10.00	10.30-11.30	11.35-12.30	2.00-4.00	4.30-5.30
	Algebra I	Real Analysis	Diff. Topology	Tutorial	UM Lectures

Daily Programme for Annual Foundation School-II					
Time	9.00-10.00	10.30-11.30	11.35-12.30	2.00-4.00	4.30-5.30
	Algebra II	Complex Analysis	Alg. Topology	Tutorial	UM Lectures

Syllabus for the Annual Foundation School-I

Algebra I

Modules over PIDs (8 lectures) The basic theory, structure theorem for f.g. abelian groups and canonical forms of matrices.

Galois theory (16 lectures) Separable and normal extensions, algebraically closed fields, splitting fields, Fundamental theorem of Galois theory, Galois groups of cubic and quartics, fundamental theorem of algebra, finite fields, Galois's solvability criterion, cyclotomic and abelian extensions,

Texts/References:

1. N. Jocobson, Basic Algebra I.
2. S. Lang, Algebra, 3rd edition.
3. M. Artin, Algebra.
4. Dummit and Foote, Algebra.

Real Analysis

Basics (12 lectures) Measures, Integration, Normed spaces, Baire category theorem. Open mapping theorem, Closed graph theorem, Uniform boundedness theorem.

Introduction to Fourier Analysis(6 lectures)

Basic theory of ordinary differential equations (6 lectures) Existence of local solutions for first order systems, maximal time of existence, finite time blow-up, global solutions. Gronwall inequality, Continuous dependence on initial data and on the vector field on bounded intervals. Examples of linear systems, Fundamental solutions.

Texts/References

1. Real Analysis by G.B.Folland, John Wiley, 1999.
2. Real And Complex Analysis by W. Rudin, McGraw Hill, 1987.

Differential geometry and topology

Basics (4 lectures) Smooth maps, bump functions, smooth partition of unity. Inverse and implicit function theorems.

Basic Theory of manifolds (5 lectures) Manifolds, tangent space, immersions submersions. Regular and critical values, Sard's theorem.

Classification (5 lectures) Transversality. Embedding manifolds in euclidean spaces. Classification of 1-dim. manifolds. Orientability.

Intersection theory and applications (6 lectures) Normal bundle and epsilon-nbds; Brouwer's degree of a map, winding number. Brouwer's fixed point theorem, Fundamental Theorem of Algebra, Jordan-Brouwer's separation theorem.

Vector fields, Poincare-Hopf index theorem, Hopf degree theorem. **(4 lectures)**

References/Texts

1. V. Guillemin, and A. Pollack, Differential Topology.
2. A. A. Kosinski, Differential Manifolds. 138, Pure and applied Mathematics, Academic Press.
3. John Milnor, Topology from the differentiable viewpoint, Univ. Press of Virginia, Charlottesville, USA, 1965.

Syllabus for Annual Foundation School-II

Algebra II

(1) **Basic commutative algebra:** Prime ideals and maximal ideals, Zariski topology, Nil and Jacobson radicals, Localization of rings and modules, Noetherian rings, Hilbert Basis theorem, modules, primary decomposition, integral dependence, Noether normalization lemma, principal ideal theorem, Hilbert's Nullstellensatz, structure of artinian rings. **(12 lectures)**
(2) **Introduction to algebraic geometry 6 lectures**algebraic varieties, dimension, singular and nonsingular points, intersection multiplicities, Bezout's theorem. Max Noether's and Pascal's theorems, cubic curves, group law on elliptic curves.

(3) **Introduction to algebraic number theory 6 lectures**

Text/References:
1. M. F. Atiyah and I. G. Macdonald, Introduction to commutative algebra.
2. D. Eisenbud, Commutative algebra with a view towards algebraic geometry.

Complex analysis

(1) Analytic functions, Path integrals, Winding number, Cauchy integral formula and consequences. Hadamard gap theorem, Isolated singularities, Residue theorem, Liouville theorem.
(2) Casorati-Weierstrass theorem, Bloch-Landau theorem, Picard's theorems, Mobius transformations, Schwarz lemma, Extremal metrics, Riemann mapping theorem, Argument principle, Rouche's theorem.
(3) Runge's theorem, Infinite products, Weierstrass p-function, Mittag-Leffler expansion.

Text: Complex analysis by Murali Rao & H. Stetkaer, World Scientific, 1991

Algebraic topology

(1) Basic notion of homotopy; contractibility, deformation etc. Some basic constructions such as cone, suspension, mapping cylinder etc. fundamental group; computation for the circle. Covering spaces and fundamental group. Simplicial Complexes, CW complexes.
(2) Simplicial Complexes, CW complexes. Homology theory and applications: Simplicial homology, Singular homology, Cellular homology of CW-complexes, Jordan-Brouwer separation theorem, invariance of domain, Lefschetz fixed point theorem etc.
(3) Categories and functors; Axiomatic homology theory.

Texts/References
1. E. H. Spanier, Algebraic Topology, Tata-McGraw-Hill
2. A. Hatcher, Algebraic topology,Cambridge University Press.